Repetition

$$(\overrightarrow{CP'})_{DH} = \mathrm{M}(\overrightarrow{O^W P})_{WH} = \mathrm{M}_I \mathrm{M}_E (\overrightarrow{O^W P})_{WH} \qquad (13.42)$$

$$\mathrm{M}_E = [\mathbf{R}, \mathbf{t}],$$

- $\mathbf{M}_E$ contains:
  - the change of basis matrix **R** (that brings us into the frame of the camera)
  - the translation vector **t**, that brings us to the origin of the camera position.
  - $\mathbf{M}_E$ ”pose” information about the camera

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  - the translation vector **t**, that brings us to the origin of the camera position.
  - **M$_E$** "pose" information about the camera

$$\mathrm{M}_I = \begin{pmatrix} -f_x & 0 & c_0 \\ 0 & -f_y & r_0 \\ 0 & 0 & 1 \end{pmatrix}$$

- **M$_I$** contains:
  - The specifics of how the camera works: intrinsics
  - f_x, f_y: focal length and size pixels determine how fast parallell lines converge in the image plane
  - c_0, r_0, the center of projection

Repetition: homogenous coordinates

This is a 3by3 matrix, but for 2 Dimensional transforms in homogenous coordinates:

$$(\overrightarrow{CP'})_{DH} = Z\begin{pmatrix} c \\ r \\ 1 \end{pmatrix}, \quad \mathrm{M}_D = \begin{pmatrix} -\frac{1}{s_x} & 0 & c_0 \\ 0 & -\frac{1}{s_y} & r_0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (\overrightarrow{O'P'})_{AH} = Z\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{13.14}$$

- We use homogenous coordinates because of 2 main reasons:
  - Convenient to do pinhole perspective camera(intrinsic matrix).
  - Convenient to do translations with matrix multiplication(both intrinsic and extrinisic).

Repetition

$$(\overrightarrow{CP'})_{DH} = \mathrm{M}(\overrightarrow{O^W P})_{WH} = \mathrm{M}_I \mathrm{M}_E (\overrightarrow{O^W P})_{WH} \qquad (13.42)$$

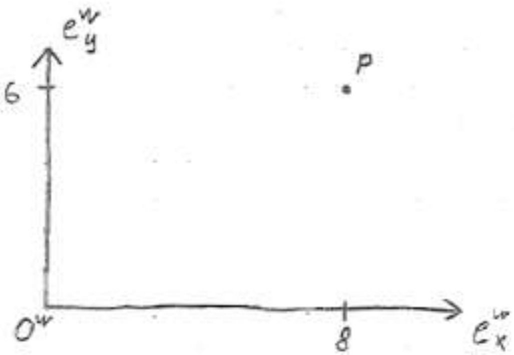
- Graphics cards accelerators work this way
  - OpenGL and Direct3D allows direct manipulation of the matrices

- In Computer graphics: world -> image

- In Computer Vision: image -> world

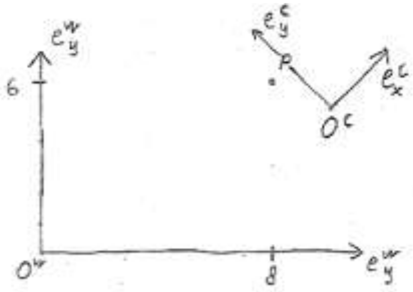
Calibration with correspondence

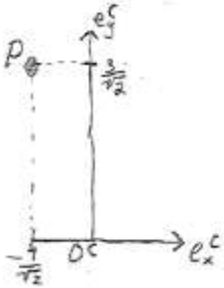
$$\boxed{(\overrightarrow{CP'})_{DH} = \mathrm{M}(\overrightarrow{O^W P})_{WH} = \mathrm{M}_I \mathrm{M}_E (\overrightarrow{O^W P})_{WH} \qquad (13.42)}$$



$$(\overrightarrow{O^w P})_w = \begin{bmatrix}8\\6\end{bmatrix} ; \quad (\overrightarrow{O^w P})_{wH} = \begin{bmatrix}8\\6\\1\end{bmatrix}$$

$$e_y^w = \begin{bmatrix}0\\1\end{bmatrix} ; \quad e_x^w = \begin{bmatrix}1\\0\end{bmatrix}$$





$$(\overrightarrow{O^wP})_w = \begin{bmatrix}8\\6\end{bmatrix};\quad (\overrightarrow{O^wP})_{wH} = \begin{bmatrix}8\\6\\1\end{bmatrix}$$

$$e_y^w = \begin{bmatrix}0\\1\end{bmatrix};\quad e_x^w = \begin{bmatrix}1\\0\end{bmatrix}$$

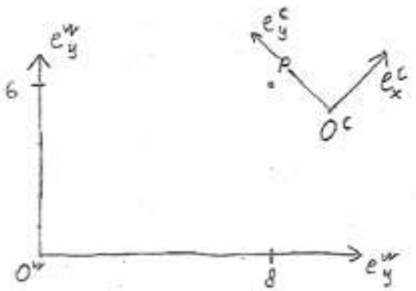
$$e_y^c = \begin{bmatrix}-1\\1\end{bmatrix}\frac{1}{\sqrt{2}};\quad e_x^c = \begin{bmatrix}1\\1\end{bmatrix}\frac{1}{\sqrt{2}}$$

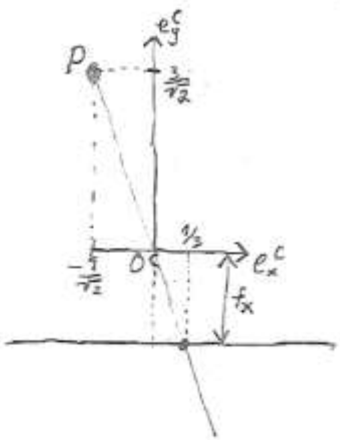
$$R^T = \left[e_x^c \;\vdots\; e_y^c\right] \Rightarrow R = \begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\frac{1}{\sqrt{2}} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$(\overrightarrow{O^wO^c})_w = \begin{bmatrix}10\\5\end{bmatrix};\quad (\overrightarrow{O^cO^w})_c = -R\cdot\begin{bmatrix}10\\5\end{bmatrix} = \begin{bmatrix}-15\\5\end{bmatrix}\frac{1}{\sqrt{2}}$$

$$M_E = \left[R \;\vdots\; (O^cO^w)_c\right] = \begin{bmatrix}1 & 1 & -15\\-1 & 1 & 5\end{bmatrix}\frac{1}{\sqrt{2}}$$

$$(\overrightarrow{O_cP})_c = M_E\,(\overrightarrow{O^wP})_{wH} = \begin{bmatrix}-1\\3\end{bmatrix}\frac{1}{\sqrt{2}}$$





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$$(\overrightarrow{O^W O^C})_W = \begin{bmatrix}10\\5\end{bmatrix}; \quad (\overrightarrow{O^C O^W})_C = -R\cdot\begin{bmatrix}10\\5\end{bmatrix} = \begin{bmatrix}-15\\5\end{bmatrix}\frac{1}{\sqrt{2}}$$

$$M_E = \left[R \;\vdots\; (O^C O^W)_C\right] = \begin{bmatrix}1 & 1 & -15\\-1 & 1 & 5\end{bmatrix}\frac{1}{\sqrt{2}}$$

$$(\overrightarrow{O_C P})_C = M_E (\overrightarrow{O^W P})_{WH} = \begin{bmatrix}-1\\3\end{bmatrix}\frac{1}{\sqrt{2}}$$

assume 1D camera, wit intrinsics:

$$M_I = \begin{bmatrix}-f_x & c_0\\0 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$$

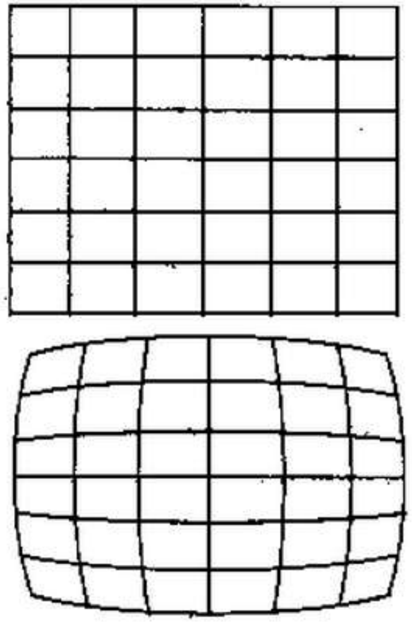
$$(\overrightarrow{CP'})_{DH} = M_I (\overrightarrow{O_C P})_C = \begin{pmatrix}1\\3\end{pmatrix}\frac{1}{\sqrt{2}} \Rightarrow \boxed{(CP')_D = \frac{1}{3}}$$

$$(\overrightarrow{CP'})_{DH} = \mathrm{M}(\overrightarrow{O^W P})_{WH} = \mathrm{M}_I \mathrm{M}_E (\overrightarrow{O^W P})_{WH} \qquad (13.42)$$

Calibration

Remember:

- The pinhole camera model is a useful model of image aquisition.
- Do not confuse it with the ”real world”, where we have more possible variations in camera aquisition, such as lens distortion



Common form of distortion in images. The pinhole camera model says all straight lines are mapped to straight lines in the projective plane – not always so in real life.

The amount of lens distortion is another intrinsic parameter that we DO NOT deal with in this course.

Many more problems comes with this simple way of modelling, but you are learning the basics in this course

Image borrowed from stackoverflow.com

Calibration

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{pmatrix} = \mathbf{M}_I \mathbf{M}_E \tag{13.43}$$

$$\mathbf{p} = (X, Y, Z, 1)^T = (\overrightarrow{O^W P})_{WH} \quad \lambda \mathbf{p}' = \lambda(c, r, 1)^T = (\overrightarrow{CP'})_{DH} \tag{13.44}$$

$$\mathbf{Mp} - \lambda \mathbf{p}' = 0 \tag{13.45}$$

$$XM_{11} + YM_{12} + ZM_{13} + M_{14} \ - c\lambda = 0 \tag{13.46}$$

$$XM_{21} + YM_{22} + ZM_{23} + M_{24} \ - r\lambda = 0 \tag{13.47}$$

$$XM_{31} + YM_{32} + ZM_{33} + M_{34} - \ \lambda = 0 \tag{13.48}$$

Calibration

$$XM_{11}+YM_{12}+ZM_{13}+M_{14}\ -c\lambda=0 \quad (13.46)$$
$$XM_{21}+YM_{22}+ZM_{23}+M_{24}\ -r\lambda=0 \quad (13.47)$$
$$XM_{31}+YM_{32}+ZM_{33}+M_{34}-\quad\lambda=0 \quad (13.48)$$

$$XM_{11}+YM_{12}+ZM_{13}+M_{14}-c(XM_{31}+YM_{32}+ZM_{33}+M_{34})=0 \quad (13.49)$$
$$XM_{21}+YM_{22}+ZM_{23}+M_{24}-r(XM_{31}+YM_{32}+ZM_{33}+M_{34})=0 \quad (13.50)$$

$$\mathbf{c}(\mathbf{p},\mathbf{p}')=(X,Y,Z,1,0,0,0,0,-cX,-cY,-cZ,-c)^T \quad (13.51)$$
$$\mathbf{r}(\mathbf{p},\mathbf{p}')=(0,0,0,0,X,Y,Z,1,-rX,-rY,-rZ,-r)^T \quad (13.52)$$

$$\mathbf{m}=(M_{11},M_{12},M_{13},M_{14},M_{21},M_{22},M_{23},M_{24},M_{31},M_{32},M_{33},M_{34})^T, \quad (13.53)$$

$$\mathbf{c}^T\mathbf{m}=0 \quad (13.54)$$
$$\mathbf{r}^T\mathbf{m}=0 \quad (13.55)$$

Calibration

$$(\overrightarrow{CP'})_{DH} = \mathcal{M}(\overrightarrow{O^W P})_{WH} = \mathcal{M}_I \mathcal{M}_E (\overrightarrow{O^W P})_{WH} \qquad (13.42)$$

Equivalent!

$$\mathbf{c}^T \mathbf{m} = 0 \qquad (13.54)$$

$$\mathbf{r}^T \mathbf{m} = 0 \qquad (13.55)$$

Calibration

Skip over the section from Eq 13.55, until 13.60 in the book (except equation 13.58 below).

Next, we consider many observations:

$$\mathcal{S} = \{\mathbf{c}^1, \mathbf{r}^1, \mathbf{c}^2, \mathbf{r}^2, \cdots, \mathbf{c}^N, \mathbf{r}^N\} \tag{13.58}$$

$$\mathrm{B}_c = \begin{pmatrix} \mathbf{c}^{1T} \\ \mathbf{c}^{2T} \\ \vdots \\ \mathbf{c}^{NT} \end{pmatrix}, \qquad \mathrm{B}_r = \begin{pmatrix} \mathbf{r}^{1T} \\ \mathbf{r}^{2T} \\ \vdots \\ \mathbf{r}^{NT} \end{pmatrix}. \qquad \mathrm{B} = \begin{pmatrix} \mathrm{B}_c \\ \mathrm{B}_r \end{pmatrix}$$

Calibration

Skip over the section from Eq 13.55, until 13.60 in the book (except equation 13.58 below).

Next, we consider many observations:

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Calibration

$$
\mathrm{B} = \begin{pmatrix}
X^1 & Y^1 & Z^1 & 1 & 0 & 0 & 0 & 0 & -c^1X^1 & -c^1Y^1 & -c^1Z^1 & -c^1 \\
X^2 & Y^2 & Z^2 & 1 & 0 & 0 & 0 & 0 & -c^2X^2 & -c^2Y^2 & -c^2Z^2 & -c^2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
X^N & Y^N & Z^N & 1 & 0 & 0 & 0 & 0 & -c^NX^N & -c^NY^N & -c^NZ^N & -c^N \\
0 & 0 & 0 & 0 & X^1 & Y^1 & Z^1 & 1 & -r^1X^1 & -r^1Y^1 & -r^1Z^1 & -r^1 \\
0 & 0 & 0 & 0 & X^2 & Y^2 & Z^2 & 1 & -r^2X^2 & -r^2Y^2 & -r^2Z^2 & -r^2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & X^N & Y^N & Z^N & 1 & -r^NX^N & -r^NY^N & -r^NZ^N & -r^N
\end{pmatrix} \tag{13.61}
$$

Calibration

$$(\overrightarrow{CP'})_{DH} = \mathrm{M}(\overrightarrow{O^W P})_{WH} = \mathrm{M}_I \mathrm{M}_E (\overrightarrow{O^W P})_{WH} \qquad (13.42)$$

$$\mathbf{c}^T \mathbf{m} = 0 \qquad (13.54)$$

$$\mathbf{r}^T \mathbf{m} = 0 \qquad (13.55)$$

$$\mathcal{S} = \{\mathbf{c}^1, \mathbf{r}^1, \mathbf{c}^2, \mathbf{r}^2, \cdots, \mathbf{c}^N, \mathbf{r}^N\} \qquad (13.58)$$

$$\mathrm{B}\mathbf{m} = 0$$

Calibration

- B is of size **12 x 2n**
  - Every row is of type '**c**' or '**r**':

$$\mathbf{c}(\mathbf{p},\mathbf{p}') = (X,Y,Z,1,0,0,0,0,-cX,-cY,-cZ,-c)^T \qquad (13.51)$$
$$\mathbf{r}(\mathbf{p},\mathbf{p}') = (0,0,0,0,X,Y,Z,1,-rX,-rY,-rZ,-r)^T \qquad (13.52)$$

  - We make **N** observations…

$$\mathcal{S} = \{\mathbf{c}^1,\mathbf{r}^1,\mathbf{c}^2,\mathbf{r}^2,\cdots,\mathbf{c}^N,\mathbf{r}^N\} \qquad (13.58)$$

  - We have exactly **N c**-rows, and **N r**-rows.

As a consequence, the matrix will be square when we have **N = 6** points observed. If we have picked those points wisely, we can get a solution. 6 is thus the absolut minimum number of point correspondences we need

In practice, we want many more than 6 points for stable solutions

Calibration

- For a square matrix B, we can do eigen analysis
- A solution is found for the eigen values equal to zero

$$\mathbf{Bm} = 0$$

- However, we do not have a square B in general
- In this case, the method of singular value decomposition can be used. This is equivalent to analyzing:

$$\mathbf{B}^T\mathbf{B}$$

- Its eigenvalues and eigen-vectors will give us the same sort of information in this case, but we ”bake in” all the extra observations for better stability
- Ideally, we should get 1 eigen-value as zero, and the other 11 as much higher.
- The zero eigenvalue eigenvector is our estimate of **m**

Calibration

- Once we have **m,** we reshape it to **M,** but we immediately get into trouble.

  If $\mathbf{m}$ is a solution for $\mathbf{Bm} = 0$ so is $\gamma\mathbf{m}$, for a real valued $\gamma \neq 0$

- We need to find the scalar , and we can do that by (see ch. 13, around the point of Equation 13.64, but beware of typos).

$$\gamma = \text{Sign}\,(t_z)\sqrt{M_{31}^2 + M_{32}^2 + M_{33}^2}$$

- And then recitfy before further processing:

$$\mathbf{M} \leftarrow \frac{1}{\gamma}\mathbf{M}$$

Calibration

- At this point, we found an estimate of **M**.
- This still leaves the task of sorting out the extrinsics from the extrinsics.

$$
\begin{aligned}
\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{pmatrix} &= \begin{pmatrix} -f_x & 0 & c_0 \\ 0 & -f_y & r_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R_{11} & R_{12} & R_{13} & t_X \\ R_{21} & R_{22} & R_{23} & t_Y \\ R_{31} & R_{32} & R_{33} & t_Z \end{pmatrix} \quad (13.63) \\
&= \begin{pmatrix} -f_x R_{11} + c_0 R_{31}, & -f_x R_{12} + c_0 R_{32}, & -f_x R_{13} + c_0 R_{33}, & -f_x t_X + c_0 t_Z \\ -f_y R_{21} + r_0 R_{31}, & -f_y R_{22} + r_0 R_{32}, & -f_y R_{23} + r_0 R_{33}, & -f_y t_Y + r_0 t_Z \\ R_{31}, & R_{32}, & R_{33}, & t_z \end{pmatrix}
\end{aligned}
$$

- straightforward algebra can be applied, and is messy to show. We can find the parameters though (next slide)

Calibration

- Intrinsics:

$$(M_{11}, M_{12}, M_{13}) \begin{pmatrix} M_{31} \\ M_{32} \\ M_{33} \end{pmatrix} = c_0$$

$$(M_{21}, M_{22}, M_{23}) \begin{pmatrix} M_{31} \\ M_{32} \\ M_{33} \end{pmatrix} = r_0$$

$$\sqrt{M_{11}^2 + M_{12}^2 + M_{13}^2 - c_0^2} = f_x$$
$$\sqrt{M_{21}^2 + M_{22}^2 + M_{23}^2 - r_0^2} = f_y$$