

Taking Search-based Software Testing to the Real World

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1. Search / Optimization / Machine Learning

- useful tools for improving testing!

2. No guarantees - but useful in practice

- less formal than many alternatives
- no guarantees in testing anyway

3. Good when “exact” alg missing

- more problems than specialized solutions
- search/optimization can adapt

What is SBSE?

What is SBST?

Why is it not real-world (enough)?

Examples of real-world applications:

Optimizing test case selection

Generating complex test data

Searching for diverse test suites

Who am I?

Tech Competence/Innovation, Broad knowledge SE

Consulting in telecom, aerospace industry, AI/Machine learning

Early pioneer of SBSE, Dynamic programming languages

A map of Europe and Asia with four orange dots marking specific locations. The dots are located in Sweden, Denmark, the United Kingdom, and Beijing, China. Blue text boxes are overlaid on the map, each containing text related to a location and year. The map shows country names in both Latin and their native scripts where applicable.

Chalmers 1991-

BTH 2006-

London & York
2013

Beijing 2009

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What is SBST?

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Examples of real-world applications:

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What is search-based software engineering?

Search-based software engineering

From Wikipedia, the free encyclopedia

Search-based software engineering (SBSE) applies [metaheuristic](#) search techniques such as [genetic algorithms](#), [simulated annealing](#) and [tabu search](#) to [software engineering](#) problems. Many activities in [software engineering](#) can be stated as [optimization](#) problems. [Optimization](#) techniques of [operations research](#) such as [linear programming](#) or [dynamic programming](#) are mostly impractical for large scale [software engineering](#) problems because of their [computational complexity](#). Researchers and practitioners use [metaheuristic](#) search techniques to find near-optimal or "good-enough" solutions.

Search-Based Software Engineering (SBSE)

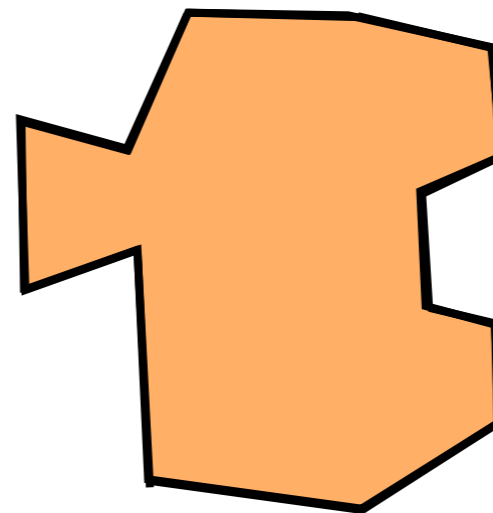
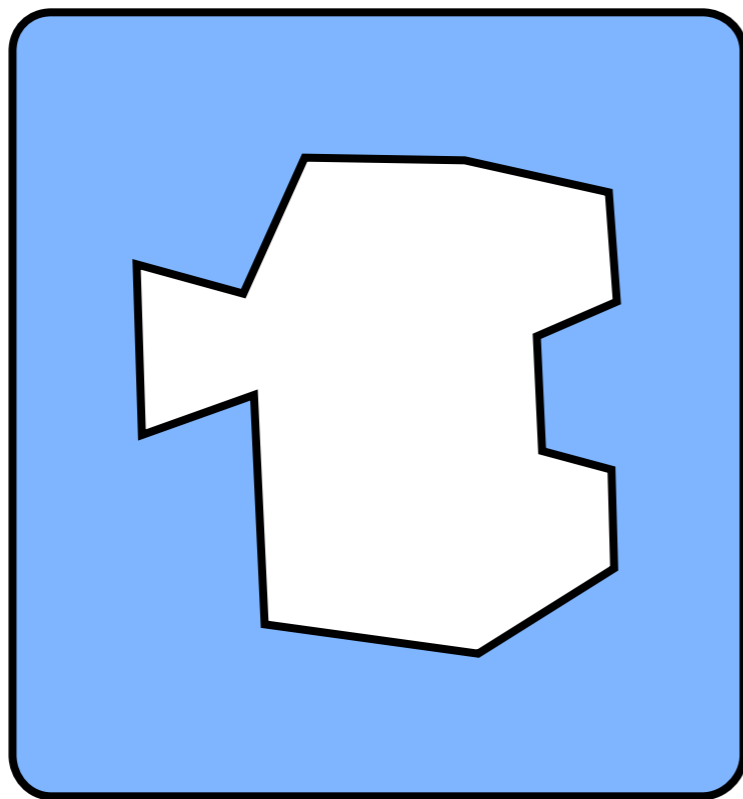
Many software engineering problems have the property that:

constructing a solution: **difficult**

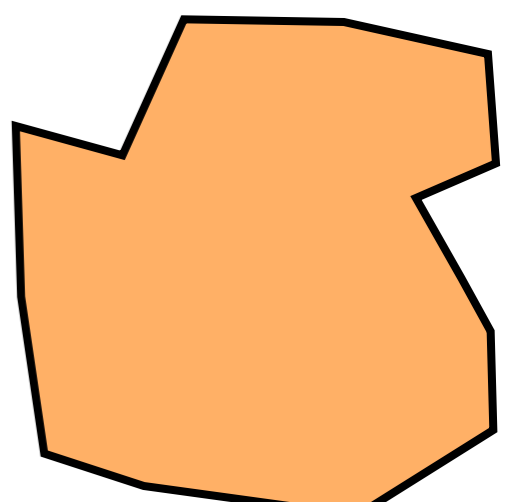
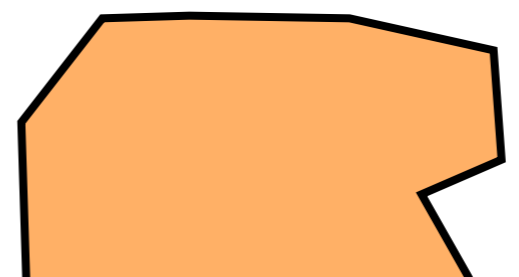
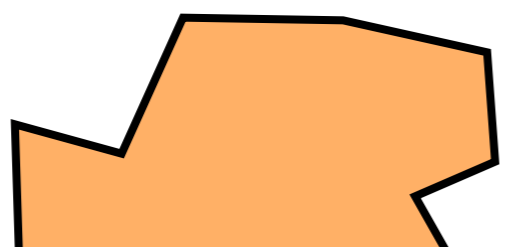
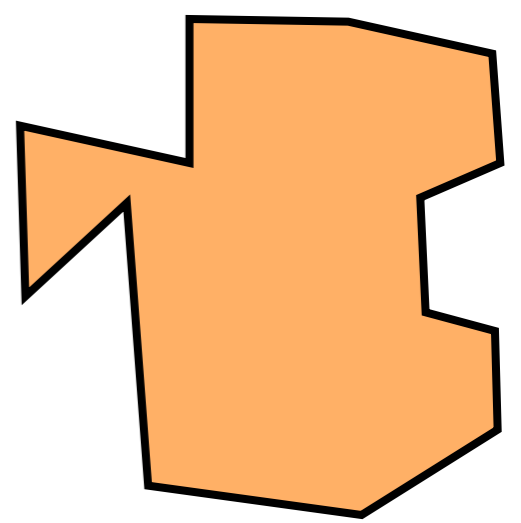
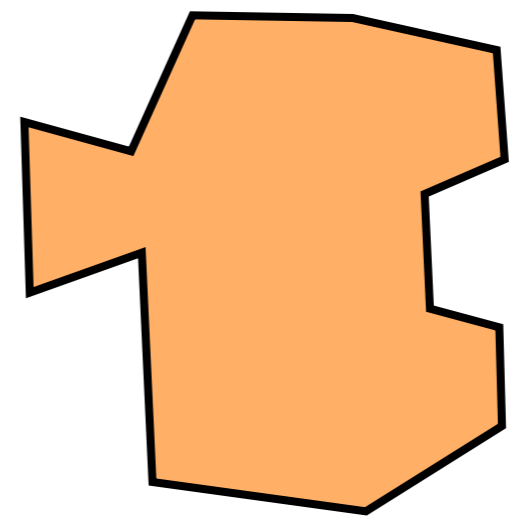
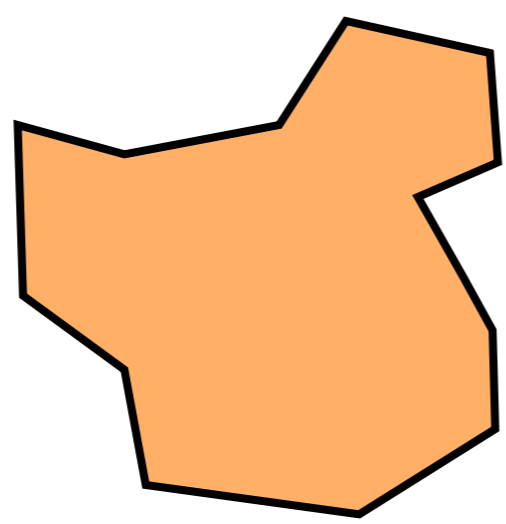
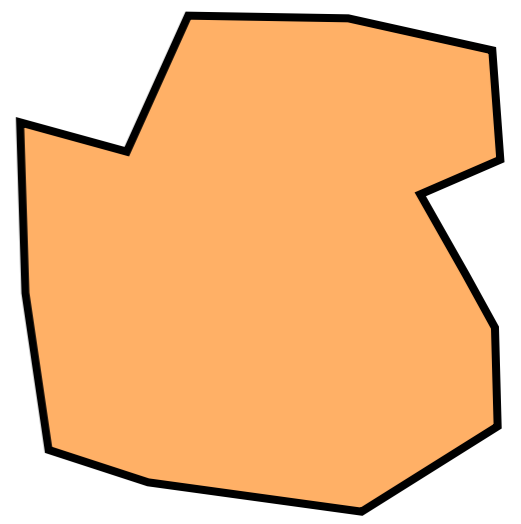
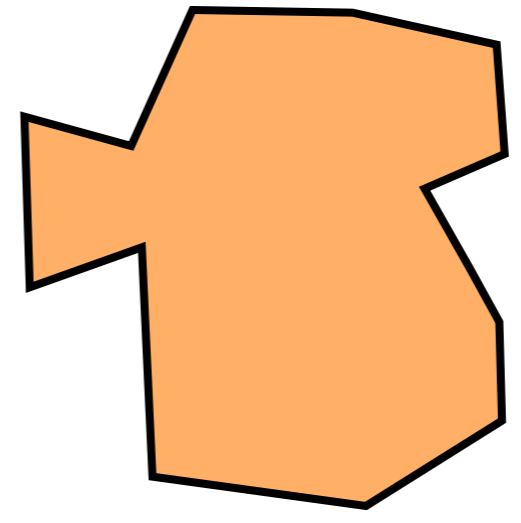
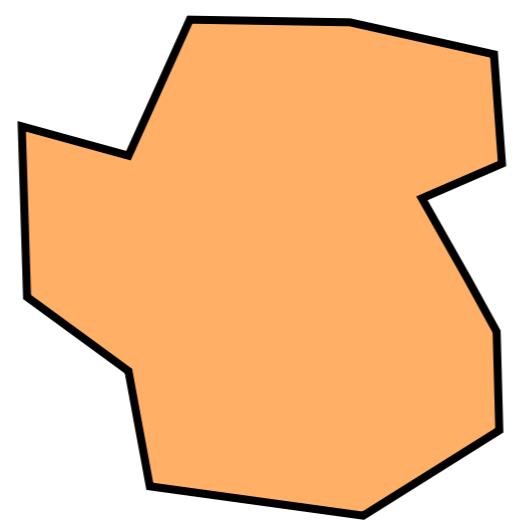
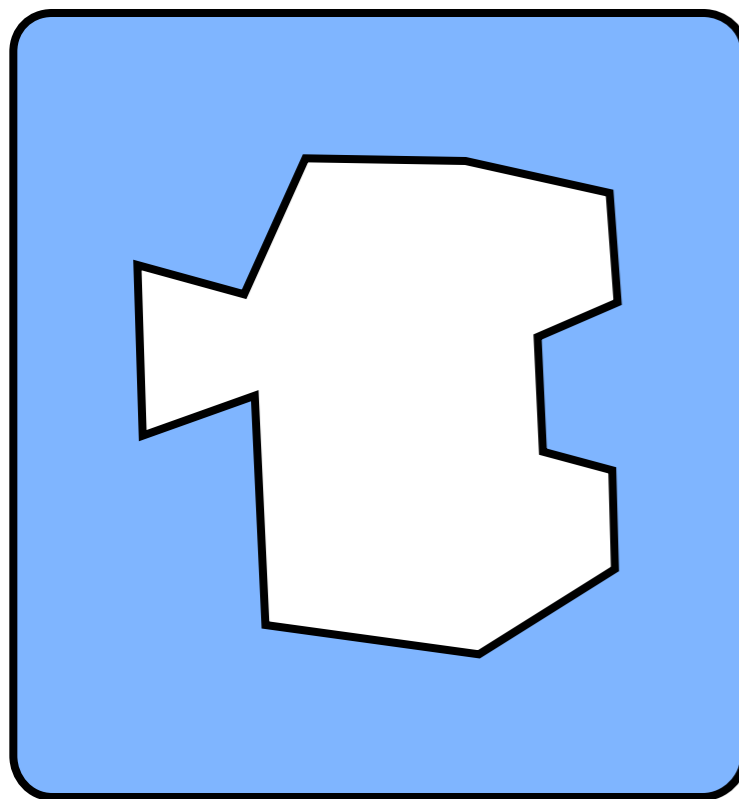
checking a potential solution: **easy**

Such tasks are amenable to solution using modern optimisation (“search”) algorithms such as **evolutionary computation**

Traditional Approach



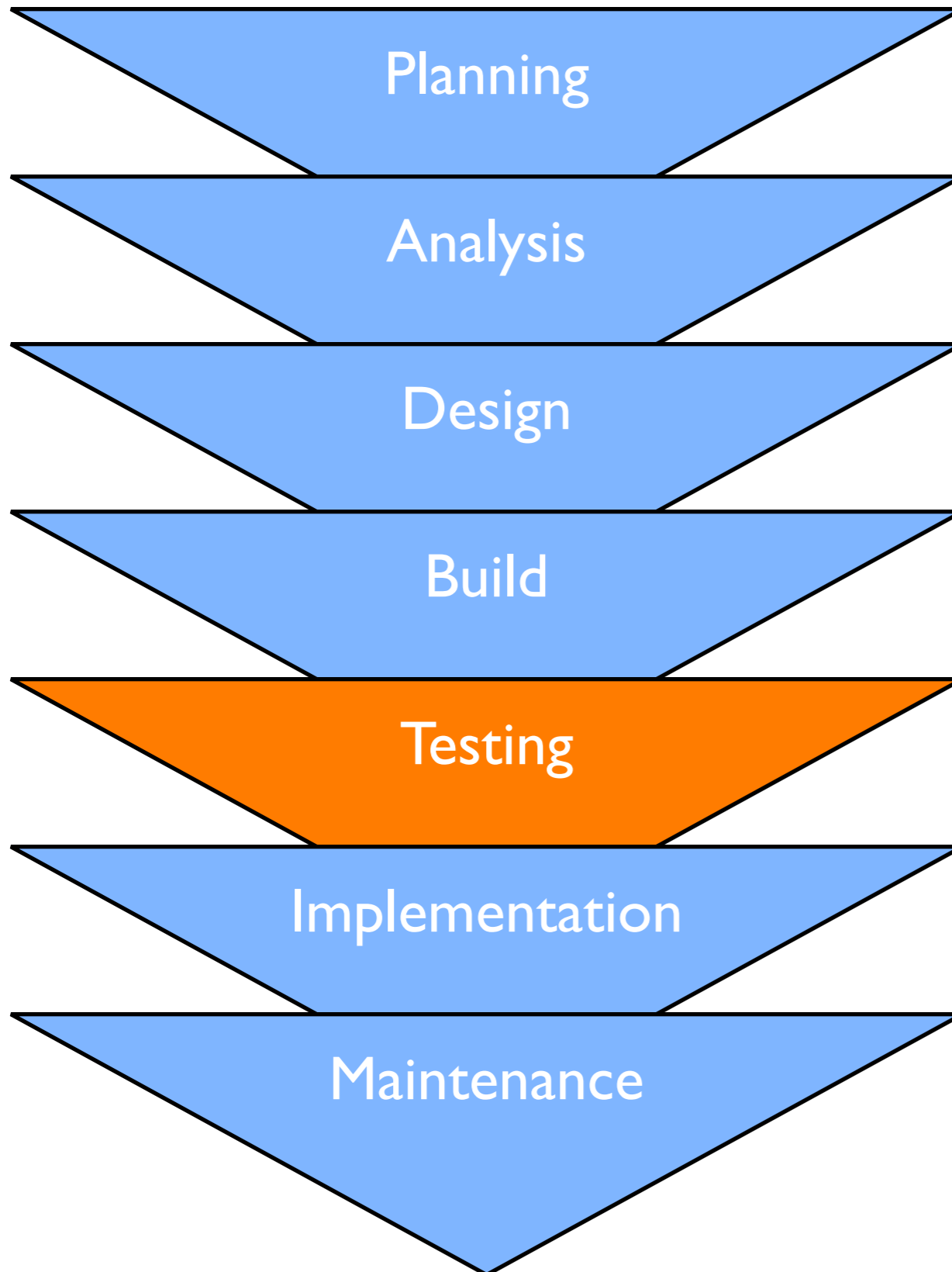
Search-Based Approach



SBSE - Advantages

- automation
- scalability
 - complex problems become tractable
 - leverages high-performance computing power
- lack of bias
 - innovation
 - more diversity
- multi-objective problems

SBSE - Applications



project planning

feature selection

real-time systems design

module clustering

protocol synthesis

concurrent software verification

algorithm construction

code refactoring

test data generation

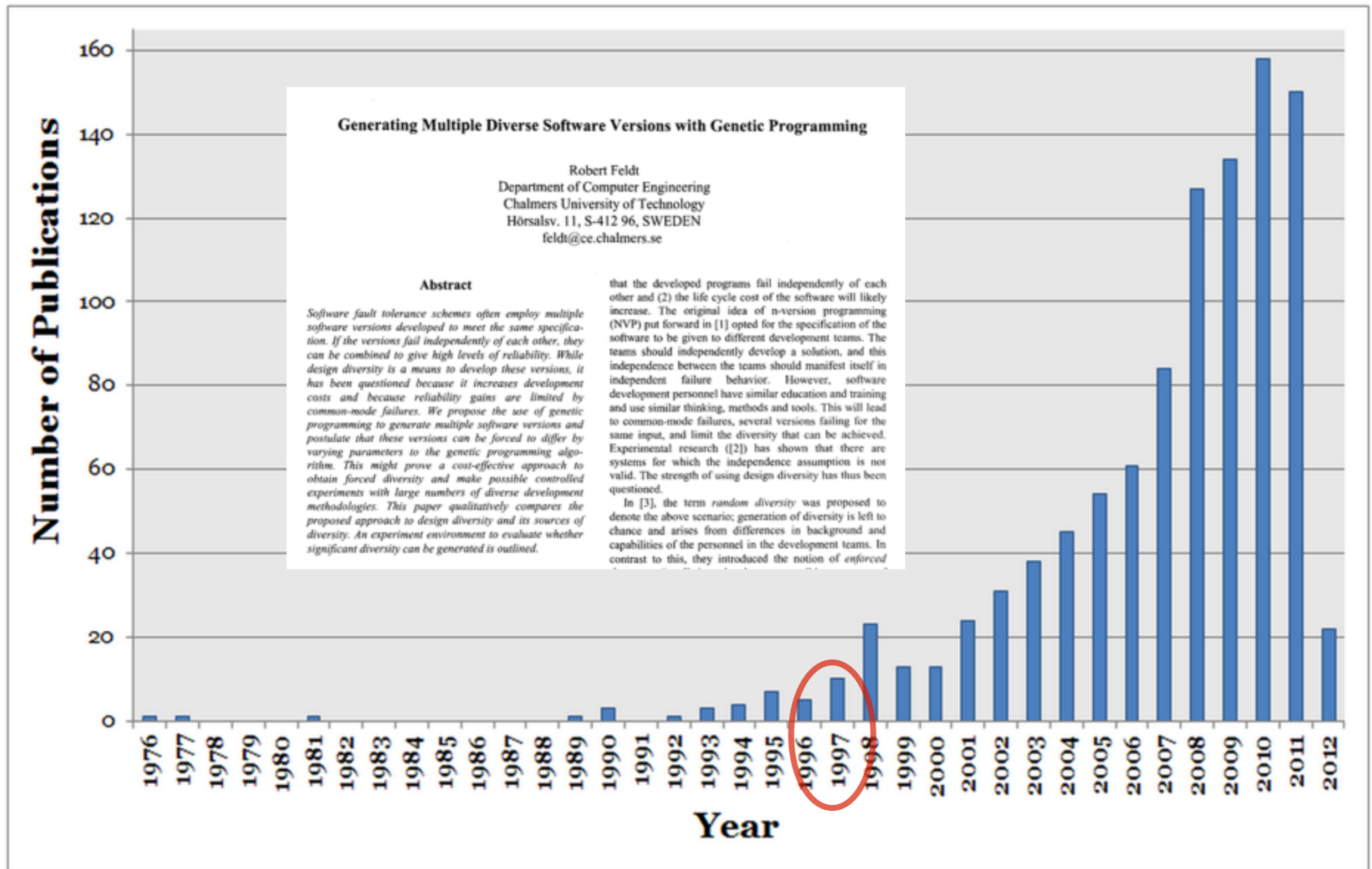
test case selection

test case prioritisation

system tuning

automated bug fixing

Why is interest in SB/ML-SE growing?



Why is interest in SB/ML-SE growing?



Bonus:
SB/ML algorithms are
embarrassingly parallel
→ good fit to modern computers

The world's top supercomputer: the Tihane-2.  Jack Dongarra

Exchange human time with CPU power

Good enough solutions without specifying details

More problems than time to find specific algorithm

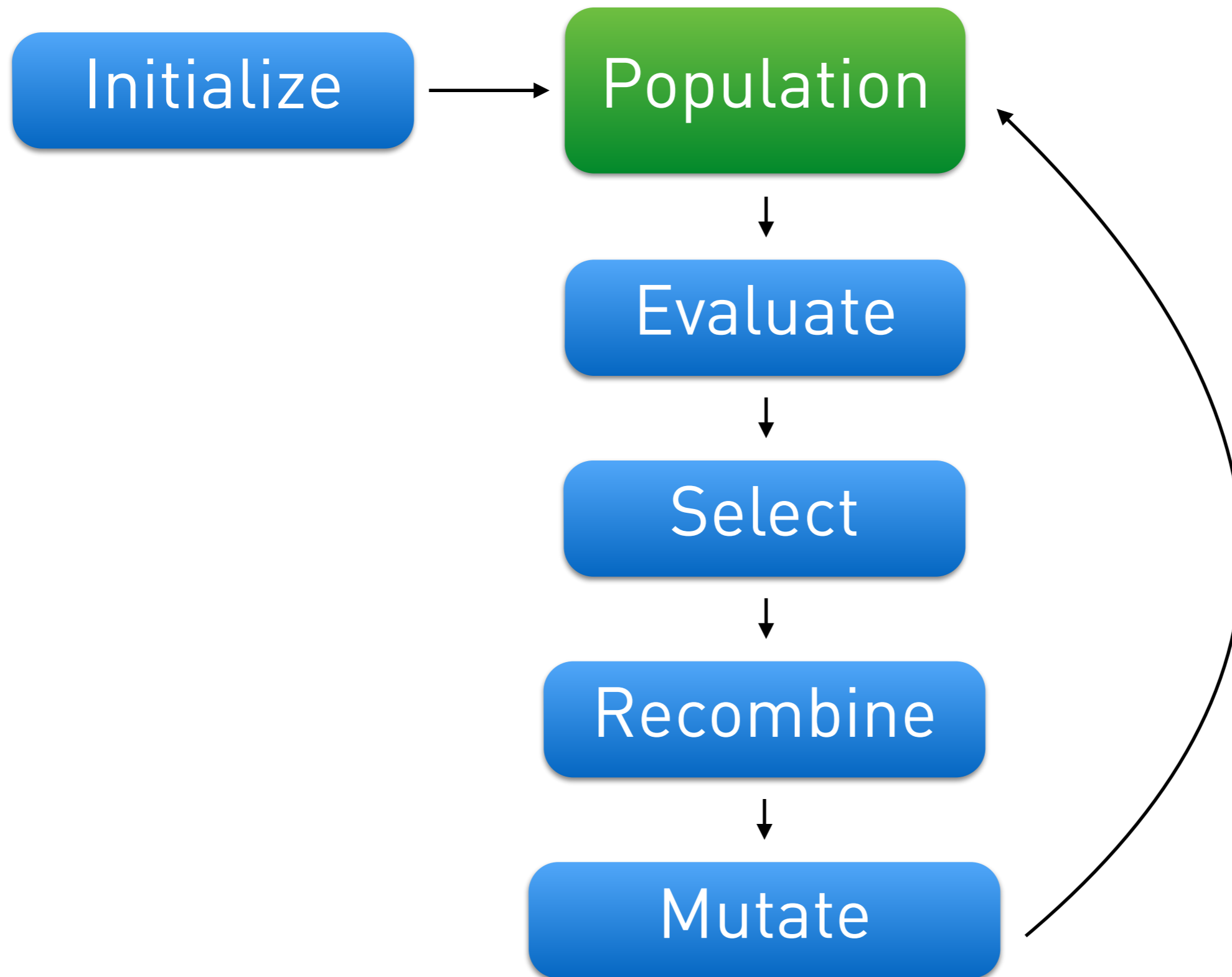
Subfield of Stochastic optimization:

“algorithms that apply some degree of randomness to find (more) optimal solutions to hard problems”

Apply for “I-Know-it-when-I-see-it” problems:

- You lack alg to find optimal solution
- You can't brute-force search
- You CAN score how good a candidate is or
- You CAN score which candidate is better

Example algorithm: Genetic Algorithm



But there are so many other search/opt algorithms!

Simulated Annealing

Hill-climbing

CMA-ES

Differential evolution

Gradient descent

Newton's method

Nesterov's method

What is SB/ML SE?

Illustration

Original



Best



Evolving



97.42% Fitness

8451 Improvements

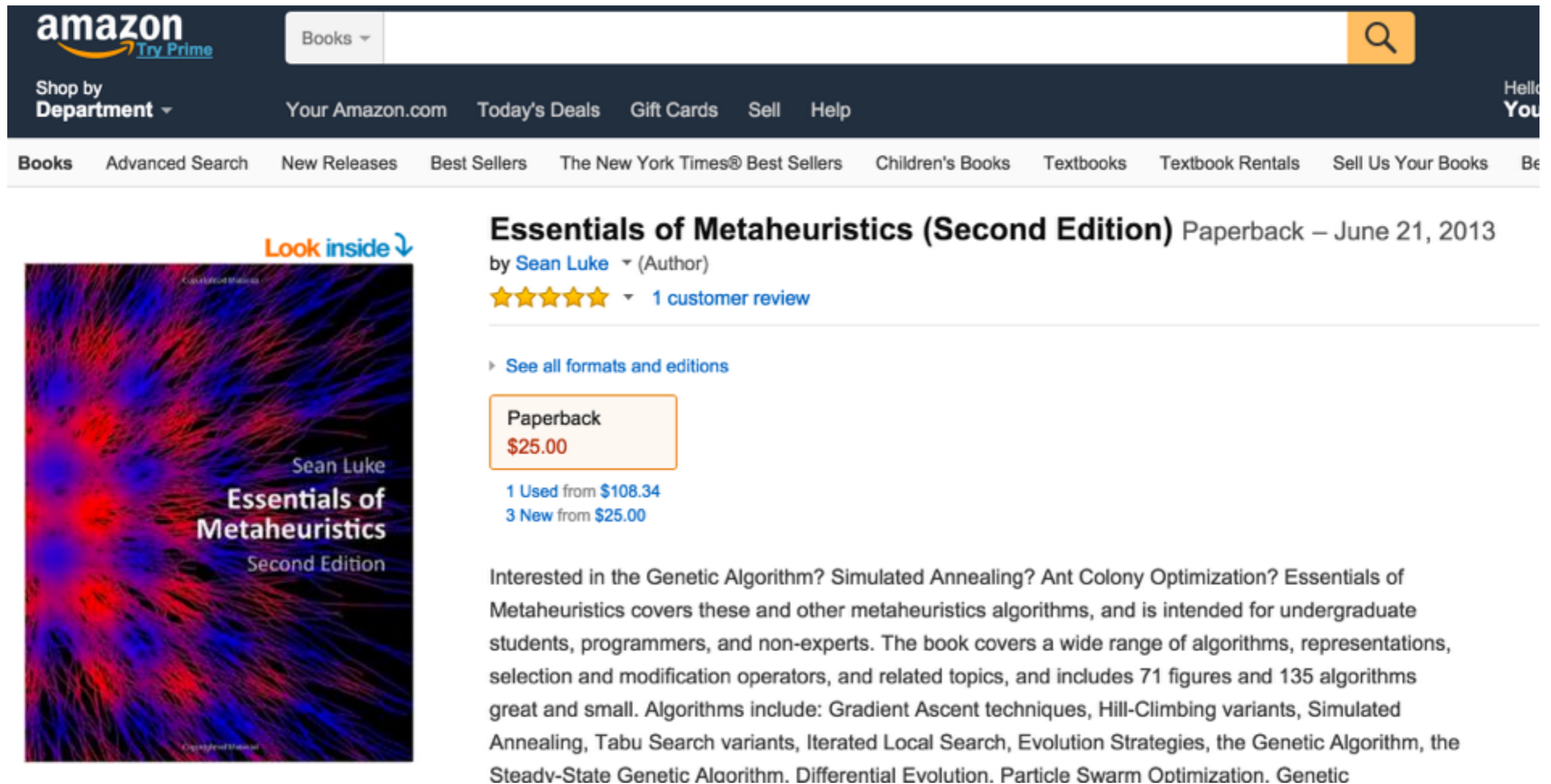
2887721 Mutations

Stop

20h 0m 52s Elapsed time

Learning more about “meta-heuristics”

<https://cs.gmu.edu/~sean/book/metaheuristics/Essentials.pdf>



The image shows a screenshot of the Amazon product page for the book "Essentials of Metaheuristics (Second Edition)" by Sean Luke. The page features the Amazon logo, a search bar, and navigation links. The book cover is displayed on the left, showing a colorful, abstract pattern of red and blue lines. The title and author's name are visible on the cover. To the right of the cover, the book's title and author are listed, along with a star rating and a customer review. Below this, there is a section for "Paperback" with a price of \$25.00 and information about used and new copies. A detailed description of the book's content is provided at the bottom.

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Look inside ↴

Essentials of Metaheuristics (Second Edition) Paperback – June 21, 2013

by **Sean Luke** ▾ (Author)

★★★★★ ▾ 1 customer review

▸ [See all formats and editions](#)

Paperback
\$25.00

1 Used from \$108.34
3 New from \$25.00

Interested in the Genetic Algorithm? Simulated Annealing? Ant Colony Optimization? Essentials of Metaheuristics covers these and other metaheuristics algorithms, and is intended for undergraduate students, programmers, and non-experts. The book covers a wide range of algorithms, representations, selection and modification operators, and related topics, and includes 71 figures and 135 algorithms great and small. Algorithms include: Gradient Ascent techniques, Hill-Climbing variants, Simulated Annealing, Tabu Search variants, Iterated Local Search, Evolution Strategies, the Genetic Algorithm, the Steady-State Genetic Algorithm. Differential Evolution. Particle Swarm Optimization. Genetic

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What is Search-Based Software Testing (SBST)?

8th International Workshop on Search-Based Software Testing (SBST) 2015

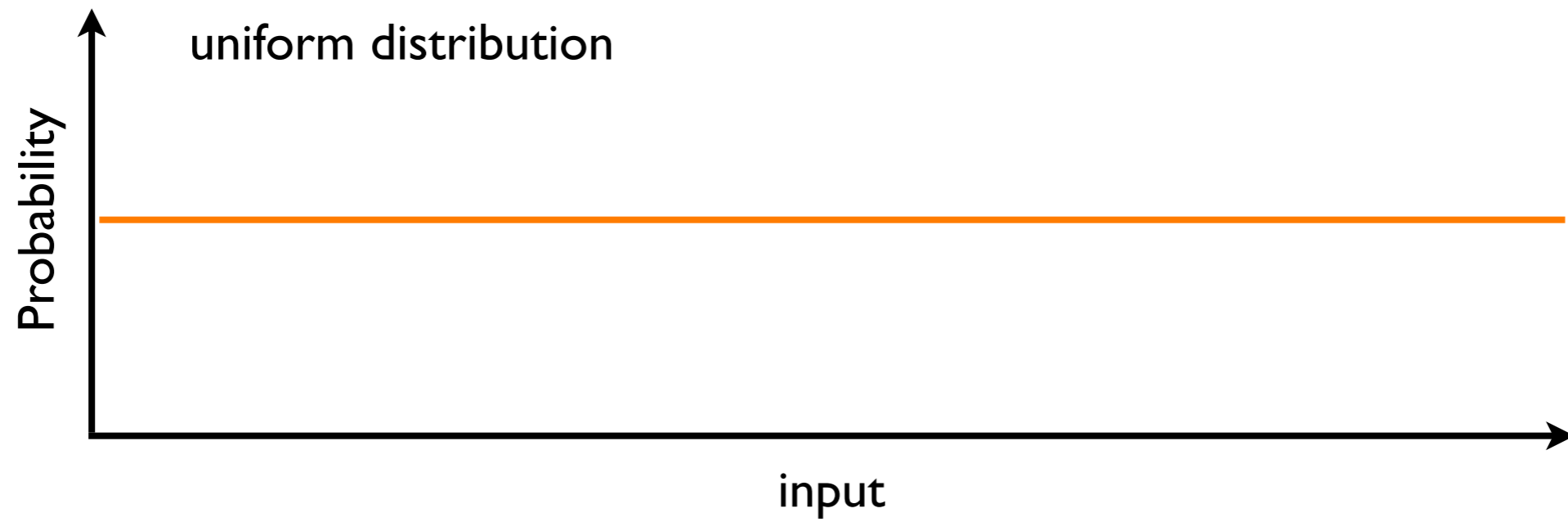
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Search-Based Software Testing (SBST) is the application of optimizing search techniques (for example, Genetic Algorithms) to solve problems in software testing. SBST is used to generate test data, prioritize test cases, minimize test suites, optimize software test oracles, reduce human oracle cost, verify software models, test service-orientated architectures, construct test suites for interaction testing, and validate real-time properties.

NEWS

Random Testing



Random Testing

```
/* 1<=a<=50, 1<=b<=20 */  
int simpleFunc(int a, int b)  
{  
    ...  
}
```

Structural Coverage Testing

```
/* 1<=a<=50, 1<=b<=20 */  
int simpleFunc(int a, int b)
```

```
int r;  
if (a<=5)
```

```
if (b>=18)
```

```
if (b<=3)
```

```
r = abs(b-19);
```

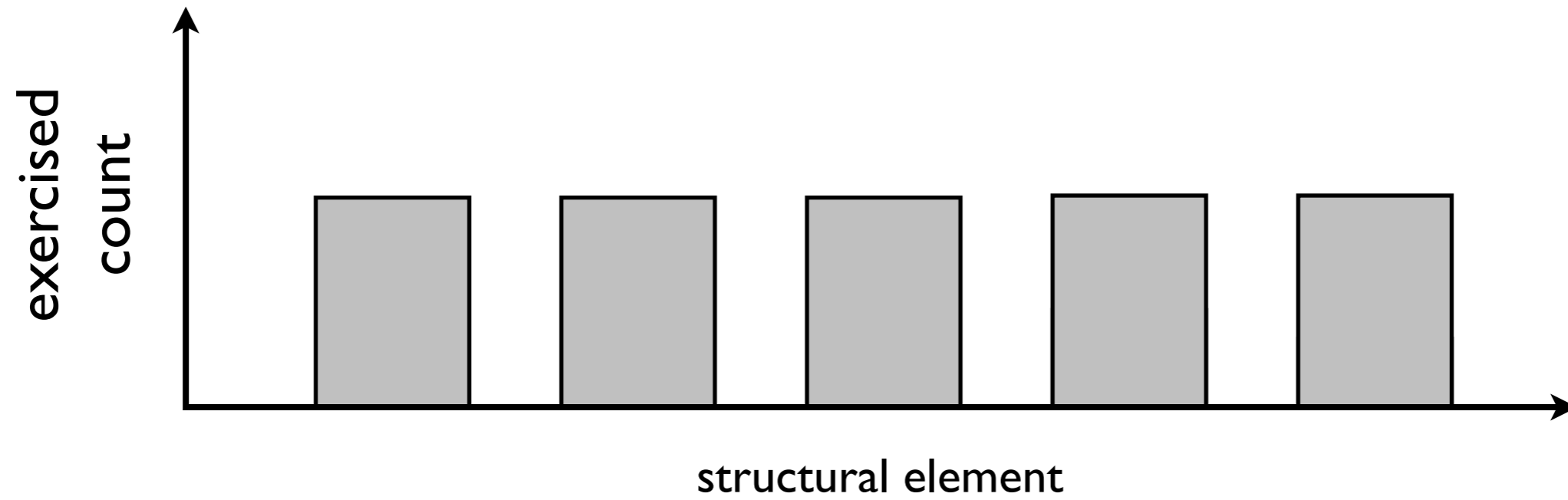
```
r = b;
```

```
r = abs(b-2);
```

```
r = 10+b;
```

```
return r;
```

Structural Coverage Testing



Structural Coverage Testing

```
/* 1<=a<=50, 1<=b<=20 */  
int simpleFunc(int a, int b)
```

```
int r;  
if (a<=5)
```

```
if (b>=18)
```

```
if (b<=3)
```

```
r = b-19;
```

```
r = b;
```

```
r = abs(b-2);
```

```
r = 10+b;
```

```
return r;
```

Structural Coverage Testing

- Difficult to **derive** test sets within specific coverage criteria
- ... but easy to **check** whether a test set satisfies a coverage criteria

Enter Search-Based Software Testing (SBST)

- traditional approach is to considers each coverage element (e.g. each branch) in turn
- find inputs that exercised the element using search methods such as
 - genetic algorithms
 - simulated annealing

Search-Based Software Testing (SBST)

Fitness function based on:

- **approach level** - how close to executing desired branch condition
- **branch distance** - how close returning the correct value for the branch condition

SBST - Approach Level

a = 6, b = 16

```
/* 1<=a<=50, 1<=b<=20 */  
int simpleFunc(int a, int b)
```

approach level = 2

```
int r;  
if (a<=5)
```

```
if (b>=18)
```

```
if (b<=3)
```

```
r = abs(b-19);
```

```
r = b;
```

```
r = abs(b-2);
```

```
r = 10+b;
```

```
return r;
```

SBST - Approach Level

a = 5, b = 16

```
/* 1<=a<=50, 1<=b<=20 */  
int simpleFunc(int a, int b)
```

```
int r;  
if (a<=5)
```

approach level = 1

```
if (b>=18)
```

```
if (b<=3)
```

```
r = abs(b-19);
```

```
r = b;
```

```
r = abs(b-2);
```

```
r = 10+b;
```

```
return r;
```

SBST - Branch Distance

$a = 5, b = 16$

```
/* 1<=a<=50, 1<=b<=20 */  
int simpleFunc(int a, int b)
```

```
int r;  
if (a<=5)
```

```
if (b>=18)
```

```
if (b<=3)
```

```
r = abs(b-19);
```

```
r = b;
```

```
r = abs(b-2);
```

```
r = 10+b;
```

```
return r;
```

branch distance
 $= |18-b| = 2$

SBST - Branch Distance

$a = 5, b = 17$

```
/* 1<=a<=50, 1<=b<=20 */  
int simpleFunc(int a, int b)
```

```
int r;  
if (a<=5)
```

```
if (b>=18)
```

```
if (b<=3)
```

```
r = abs(b-19);
```

```
r = b;
```

```
r = abs(b-2);
```

```
r = 10+b;
```

```
return r;
```

branch distance
 $= |18-b| = 1$

Broadening the Search in Search-Based Software Testing: It Need Not Be Evolutionary

Robert Feldt and Simon Poulding

Dept. of Software Engineering

Belkinge Institute of Technology, Karlskrona, Sweden

Email: robert.feldt@bth.se and simon.poulding@bth.se

Abstract—Search-based software testing (SBST) can potentially help software practitioners create better test suites using less time and resources by employing powerful methods for search and optimization. However, research on SBST has typically focused on only a few search approaches and basic techniques. A majority of publications in recent years use some form of evolutionary search, typically a genetic algorithm, or, alternatively, some other optimization algorithm inspired from nature. This paper argues that SBST researchers and practitioners should not restrict themselves to a limited choice of search algorithms or approaches to optimization. To support our argument we empirically investigate three alternatives and compare them to the de facto SBST standards in regards to performance, resource efficiency and robustness on different test data generation problems: classic algorithms from the optimization literature, bayesian optimization with gaussian processes from machine learning, and nested monte carlo search from game playing / reinforcement learning. In all cases we show comparable and sometimes better performance than the current state-of-the-SBST-art. We conclude that SBST researchers should consider a more general set of solution approaches, more consider combinations and hybrid solutions and look to other areas for how to develop the field.

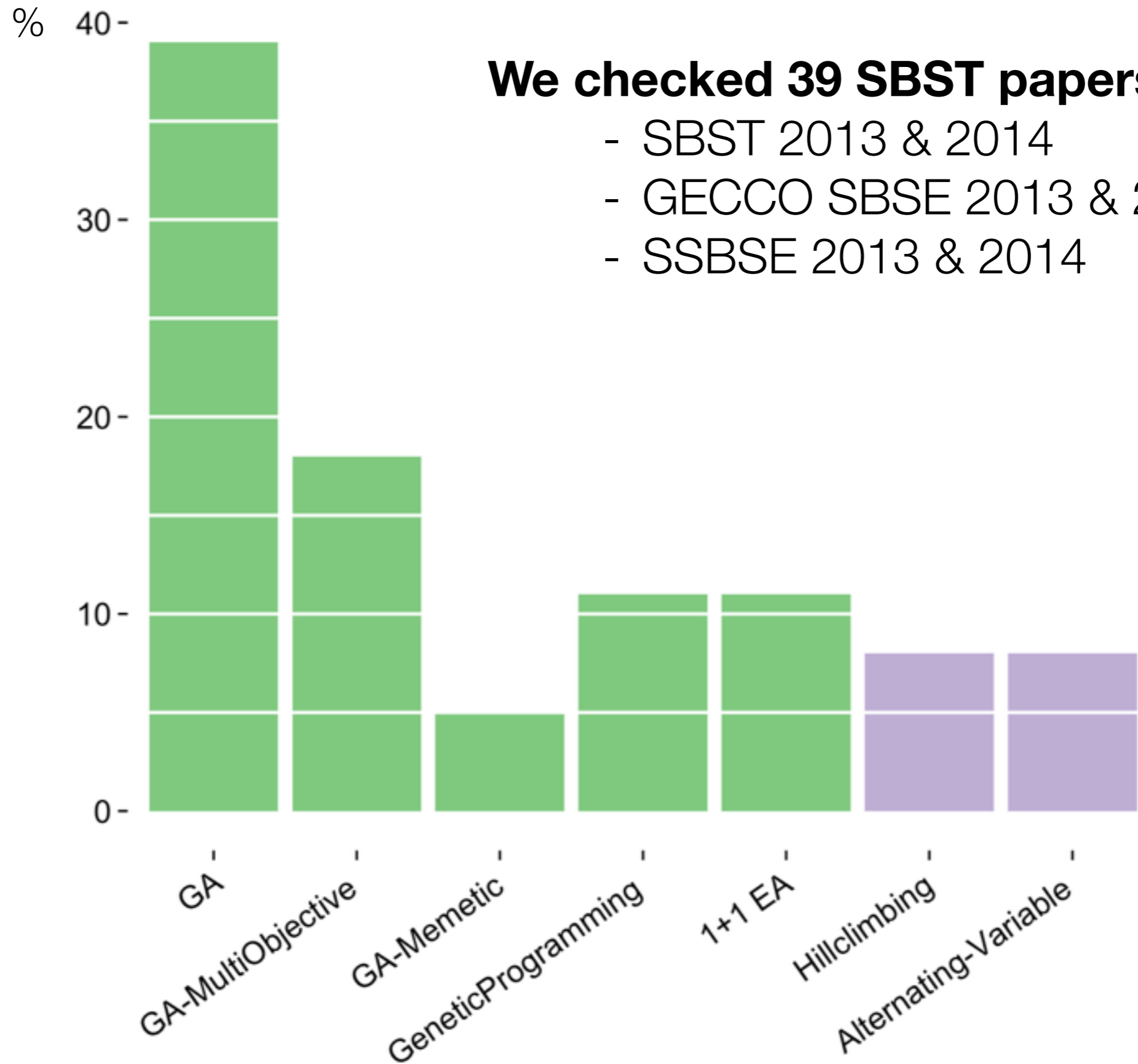
I. INTRODUCTION

The term Search-Based Software Testing (SBST) describes a number of powerful methods that permit practitioners to

published at these venues in 2013 and 2014. Two-thirds of the papers—26 out of 39—applied an evolutionary algorithm, of which 23 applied a Genetic Algorithm (GA): 15 as a standard GA, 2 as a GA-based memetic algorithm, and 7 as a multi-objective GA (the majority using NSGA-II)¹. The next most-frequently applied algorithms were Genetic Programming (4 papers), (1+1) EA (4 papers), hill-climbing (3 papers), and alternating variable methods (3 papers). Our analysis suggests that evolutionary search, and GAs in particular, are the algorithms of choice for both single- and multi-objective problems in SBST.

We offer a number of explanations for this prevalence of GAs as the search technique. GAs can be applied to a wide range of problem classes and typically find solutions with acceptably good quality. This wide applicability permits us, as researchers, to re-use the knowledge gained in applying GAs to one testing problem when solving subsequent problems. In addition, there is a great deal of active research in GAs that can guide their application to testing problems, and this research is typically disseminated in a form that is readily-accessible to us. In contrast, the research on classic optimization algorithms is often described for fellow mathematicians and may be less accessible.

But EA's and GA's are used in 60-80% of papers



What is SBSE?

What is SBST?

Why is it not real-world (enough)?

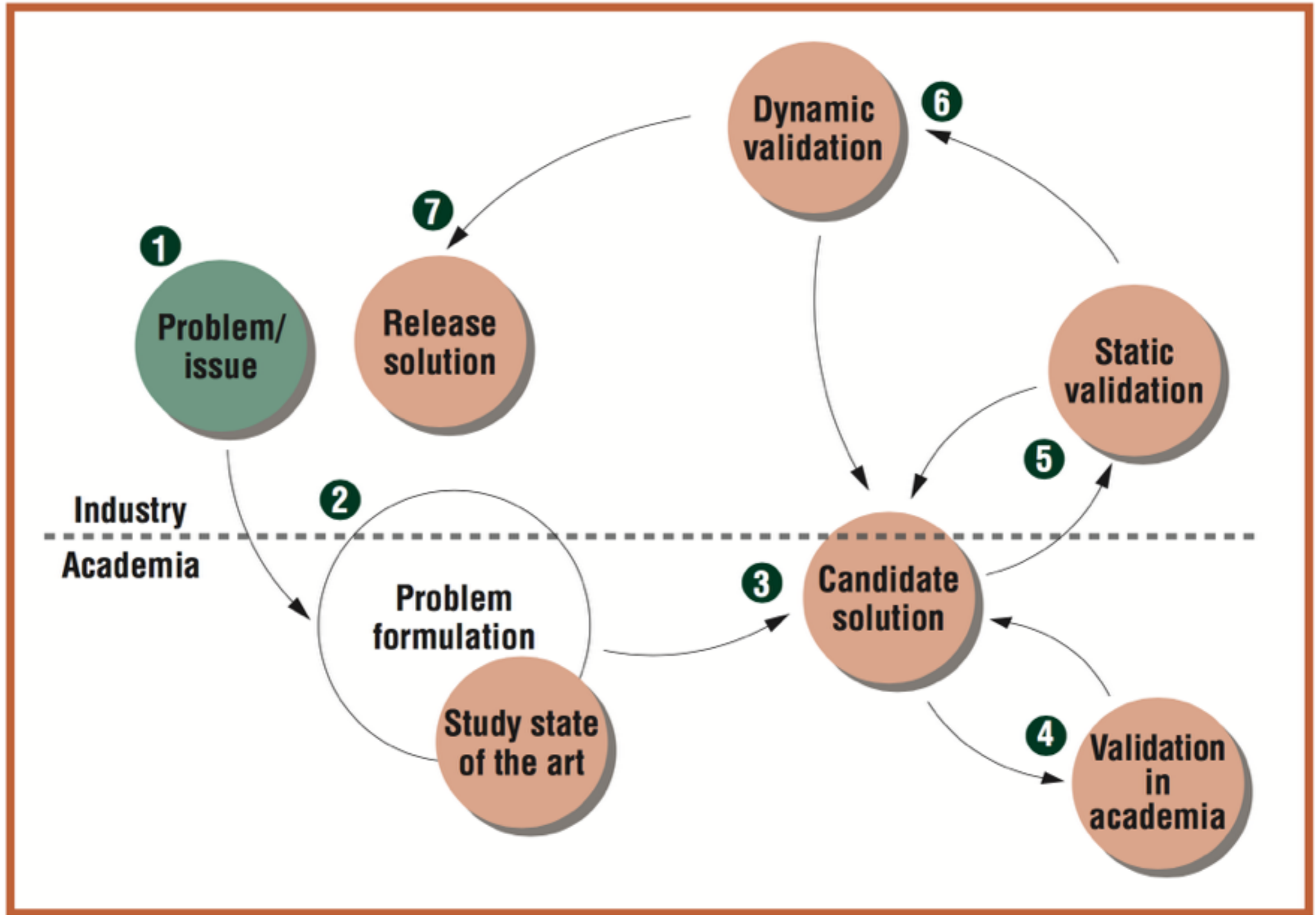
Examples of real-world applications:

Optimizing test case selection

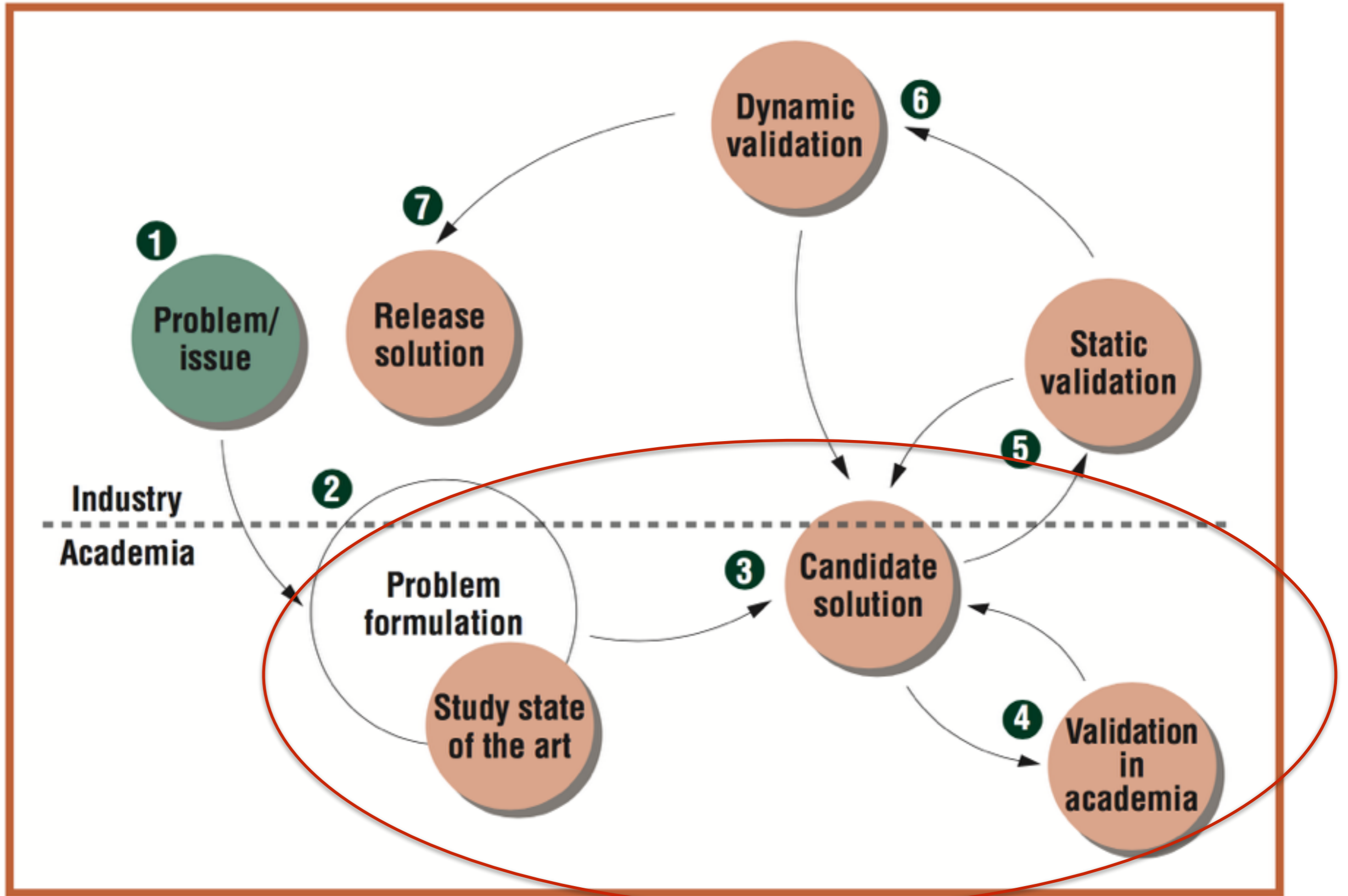
Generating complex test data

Searching for diverse test suites

Technology Transfer Model



SBST has stayed mostly within academia!



So what is SBST missing?

Application to industrial-scale systems

Not generating only numbers

Interfaces: Interaction, Visualisation

What is SBSE?

What is SBST?

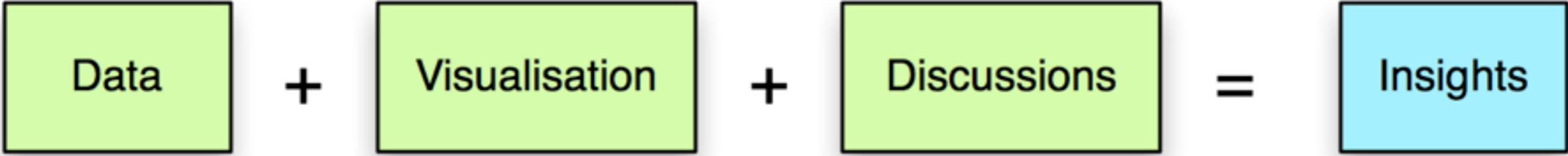
Why is it not real-world (enough)?

Examples of real-world applications:

Optimizing test case selection

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TEST START TIME

TEST CASE

SYSTEM VERSION

OUTCOME

2013-09-04 04:17:12

Login non existing user

1.32 - Build 3476

PASS

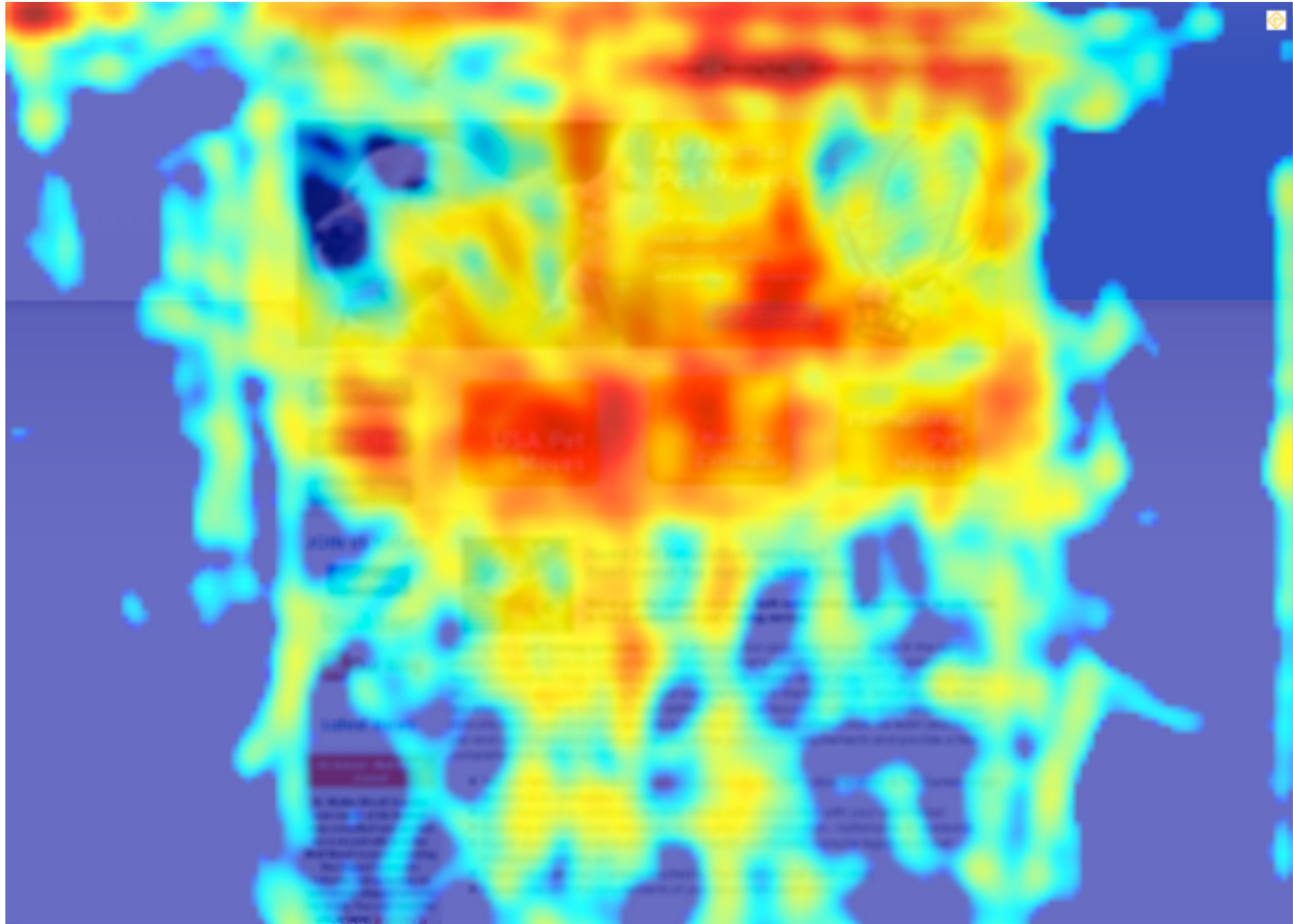
2013-09-04 04:17:12

Login existing user

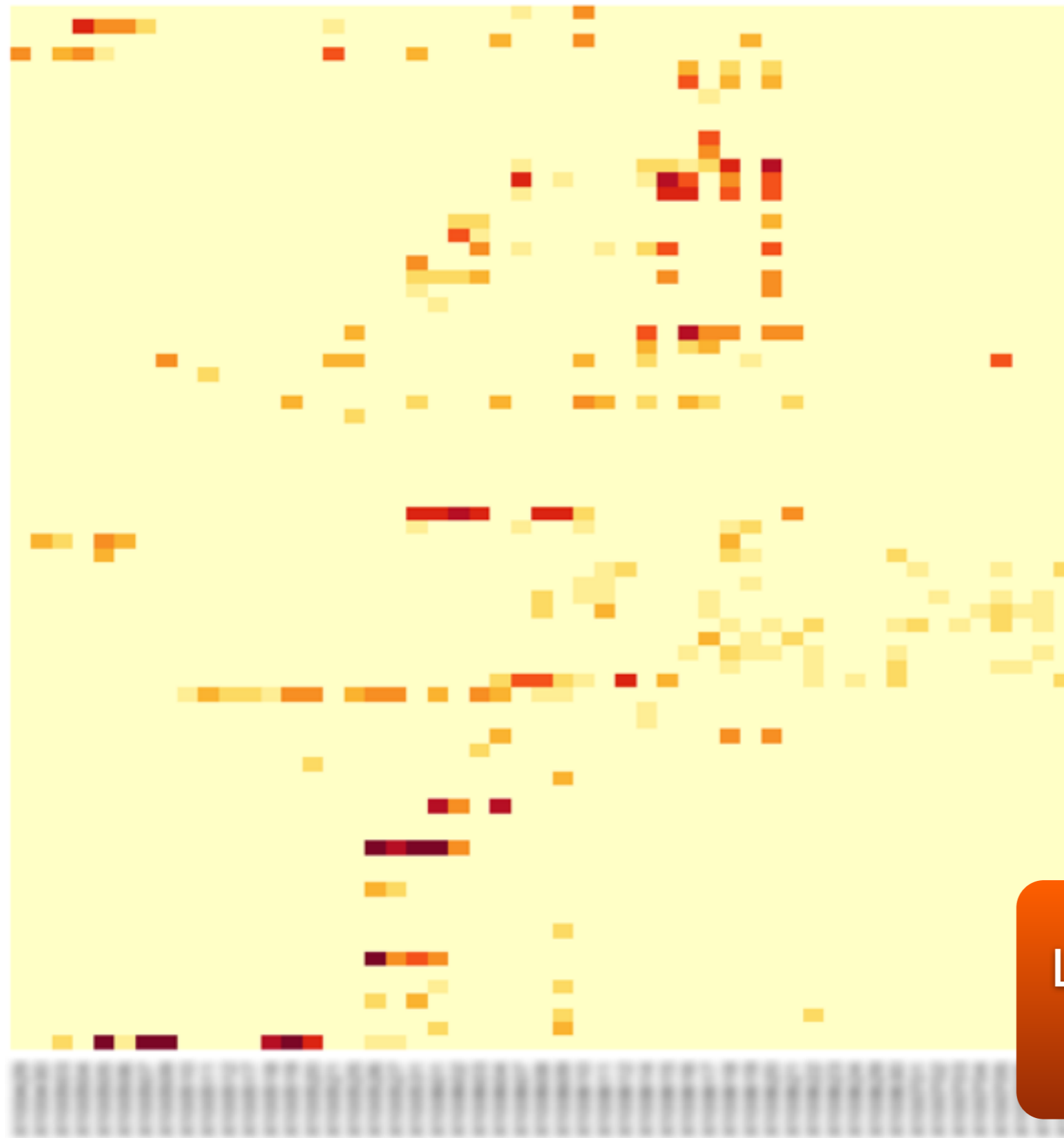
1.32 - Build 3476

FAIL

Heatmaps shows “raw” data and reveals patterns



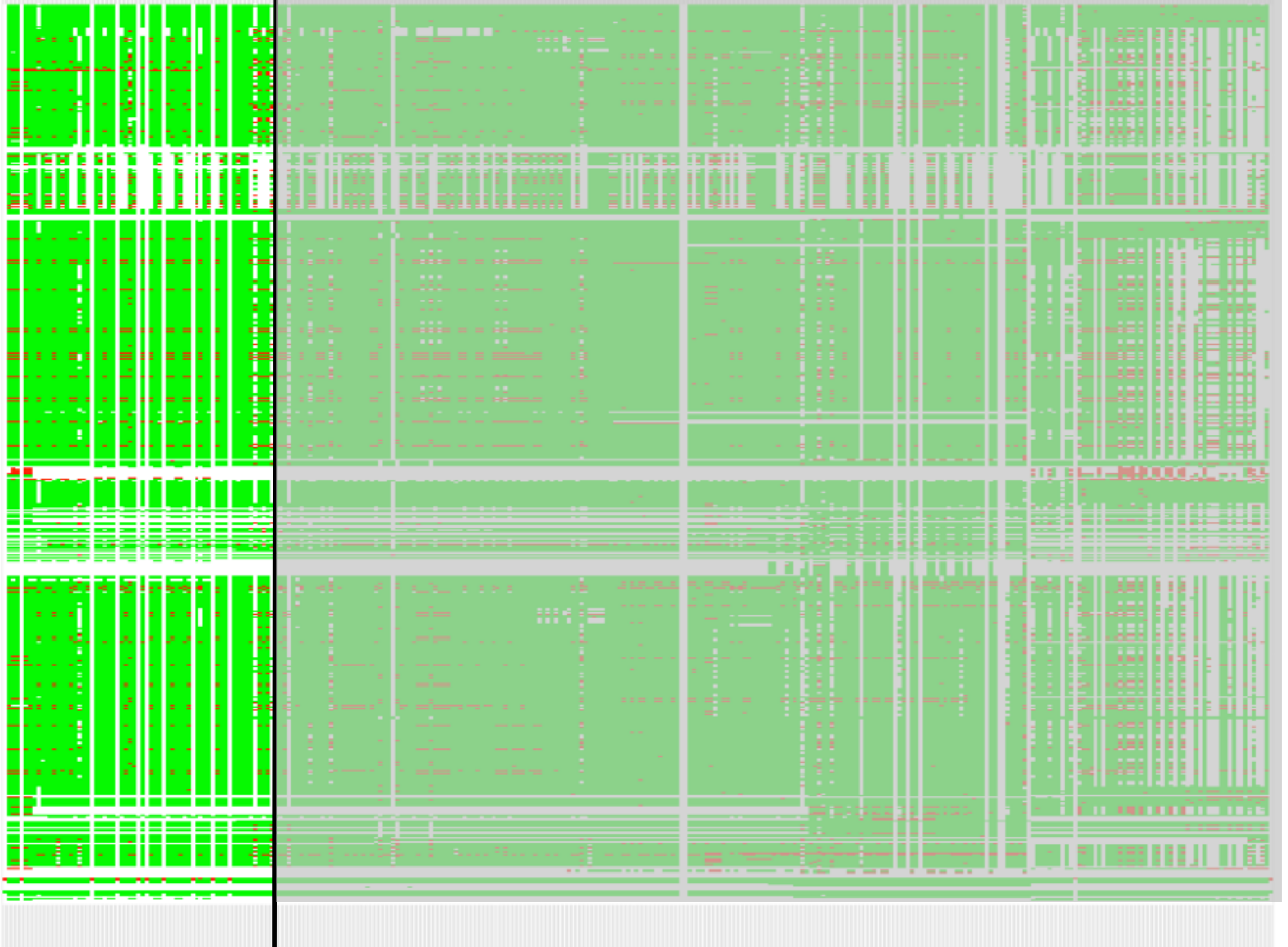
Test failures



Lack of integration traces!

Model

Model++



Most RegTest research not realistic

Requires too much (or complete) information

Source & Test code, Changes, Outcome,
Bug reports, Priorities, Severities, ...

Focus on only one criteria (Mono-objective)

In companies, things change & situations differ

Evaluated on very small examples

Scalability problems don't surface

Unclear if relevant for you

But simple modeling often give BIG benefits!

3 Types of Regression Test Selection



1. TCS = “Which n test cases should I run given P2!=P1?”

2. TSM = “Minimal test suite that gives adequate testing?”

3. TCP = “Priority order of test cases to find faults early?”

History-based Prioritization

“Use history of test outcomes to focus testing where most needed”

Example, Fazlilazadeh (2009):

Priority

Fails

Prev. Priority

Sessions since last run

$$PR_k = \alpha * \frac{f_{ck}}{e_{ck}} + \beta * PR_{k-1} + \gamma * h_k$$

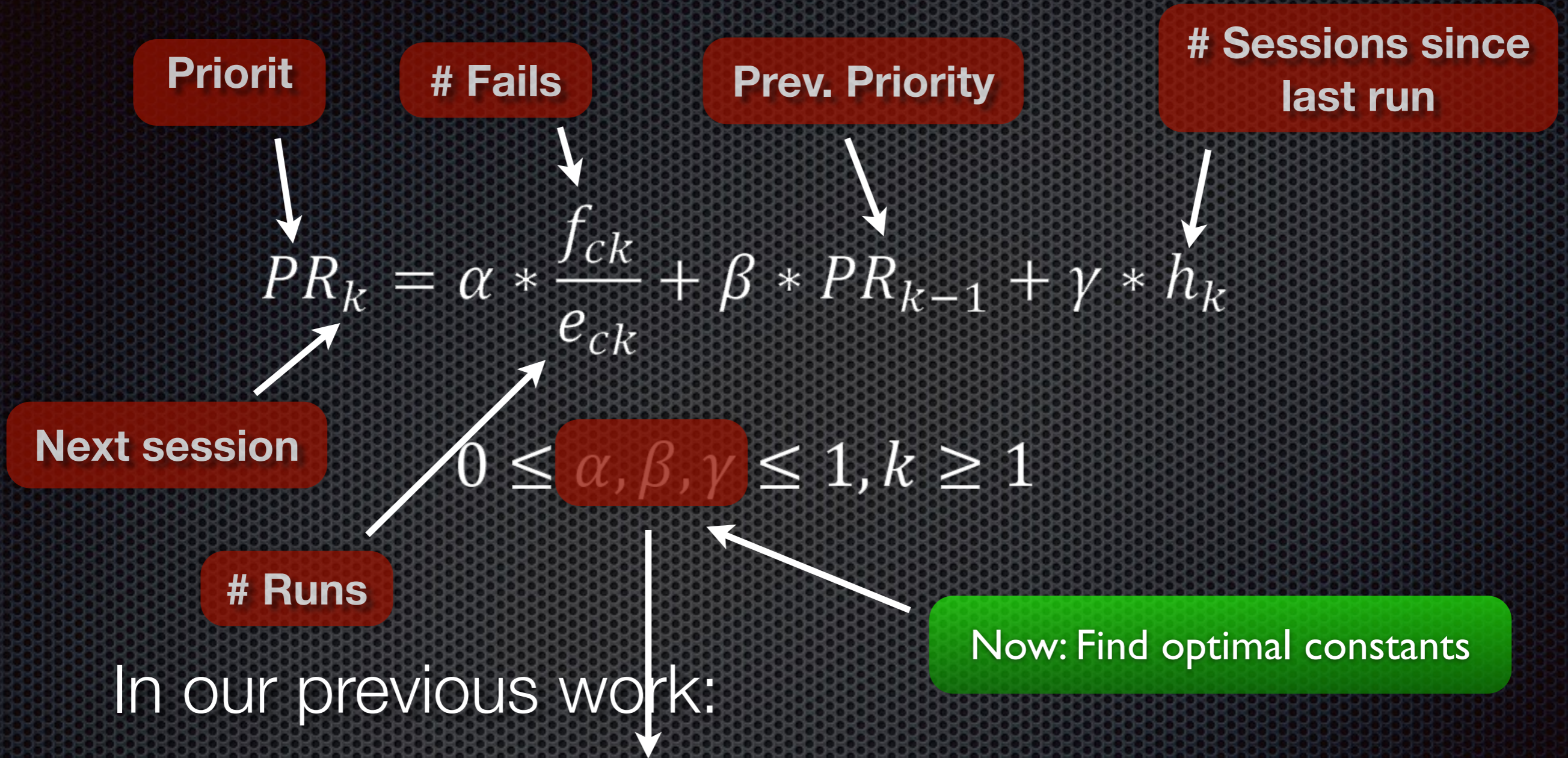
This session

$$0 \leq \alpha, \beta, \gamma \leq 1, k \geq 1$$

Runs

How to optimize history-based reg testing?

Optimizing Fazlilazadeh



	TCSR	FDR
0.3, 0.3, 0.3	60 %	86 %
0.9, 0.05, 0.1	60 %	94 %
Random sel.	60 %	59 %

Which constants give best FDR?

	Optimization	α, β, γ	TCSR	FDR
old	None, standard	Faz(0.3, 0.3, 0.3)	60 %	86 %
old	Manual testing	Faz(0.9, 0.05, 0.1)	60 %	94 %
baseline	N/A	Random sel.	60 %	59 %
new1	Simulated Annealing	None better!	60 %	94 %

Faz is robust when many test cases are selected!
(Less space for improvement)

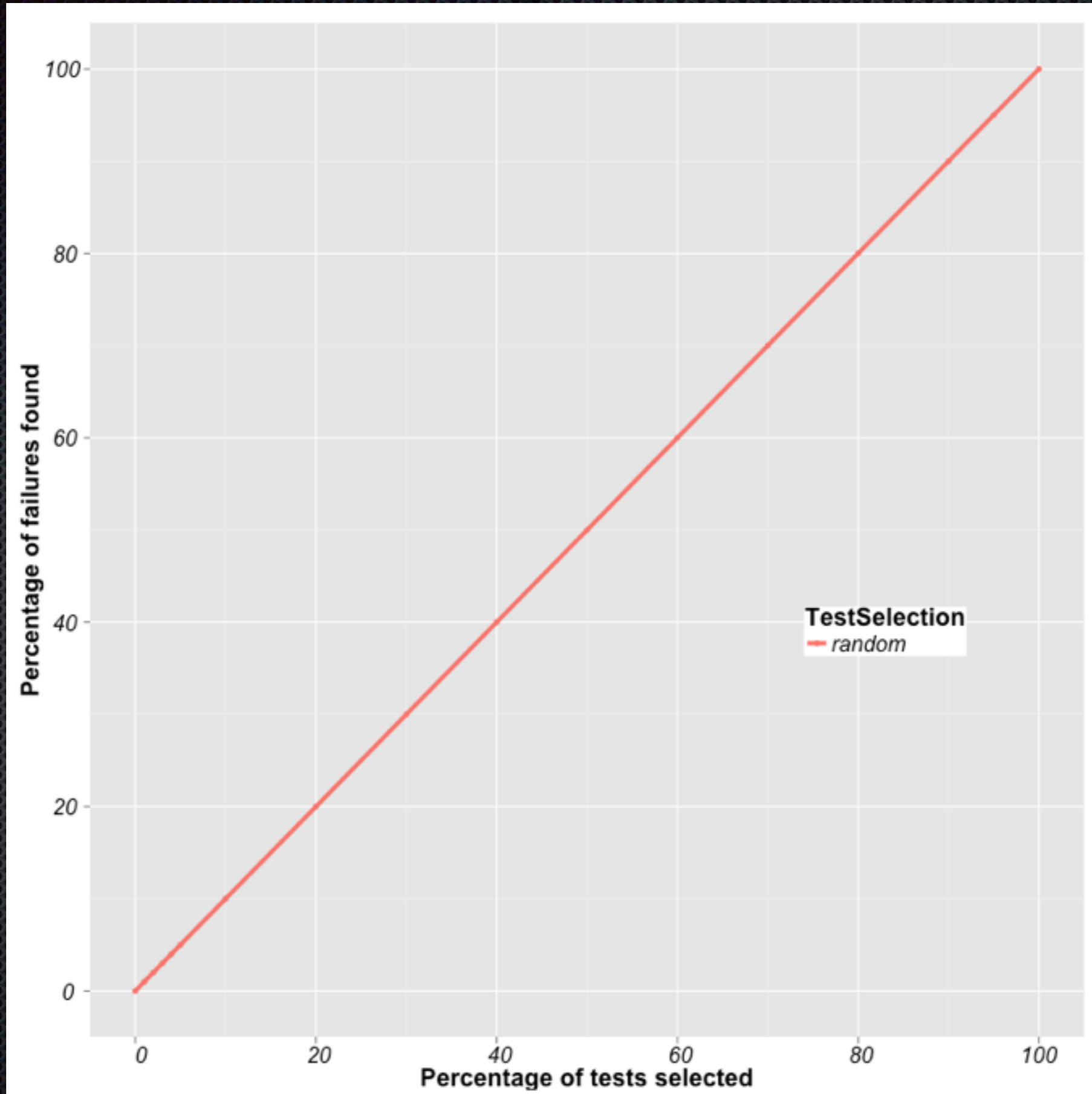
Is Faz robust for different TCSR's?

	Optimization	α, β, γ	TCSR	FDR
old	None, standard	Faz(0.3, 0.3, 0.3)	20 %	10 %
old	Manual testing	Faz(0.9, 0.05, 0.1)	20 %	57 %
baseline	NAP	Random sel.	20 %	20 %
new1	Simulated annealing	Faz(0.18, 0.47, -0.08)	20 %	89 %

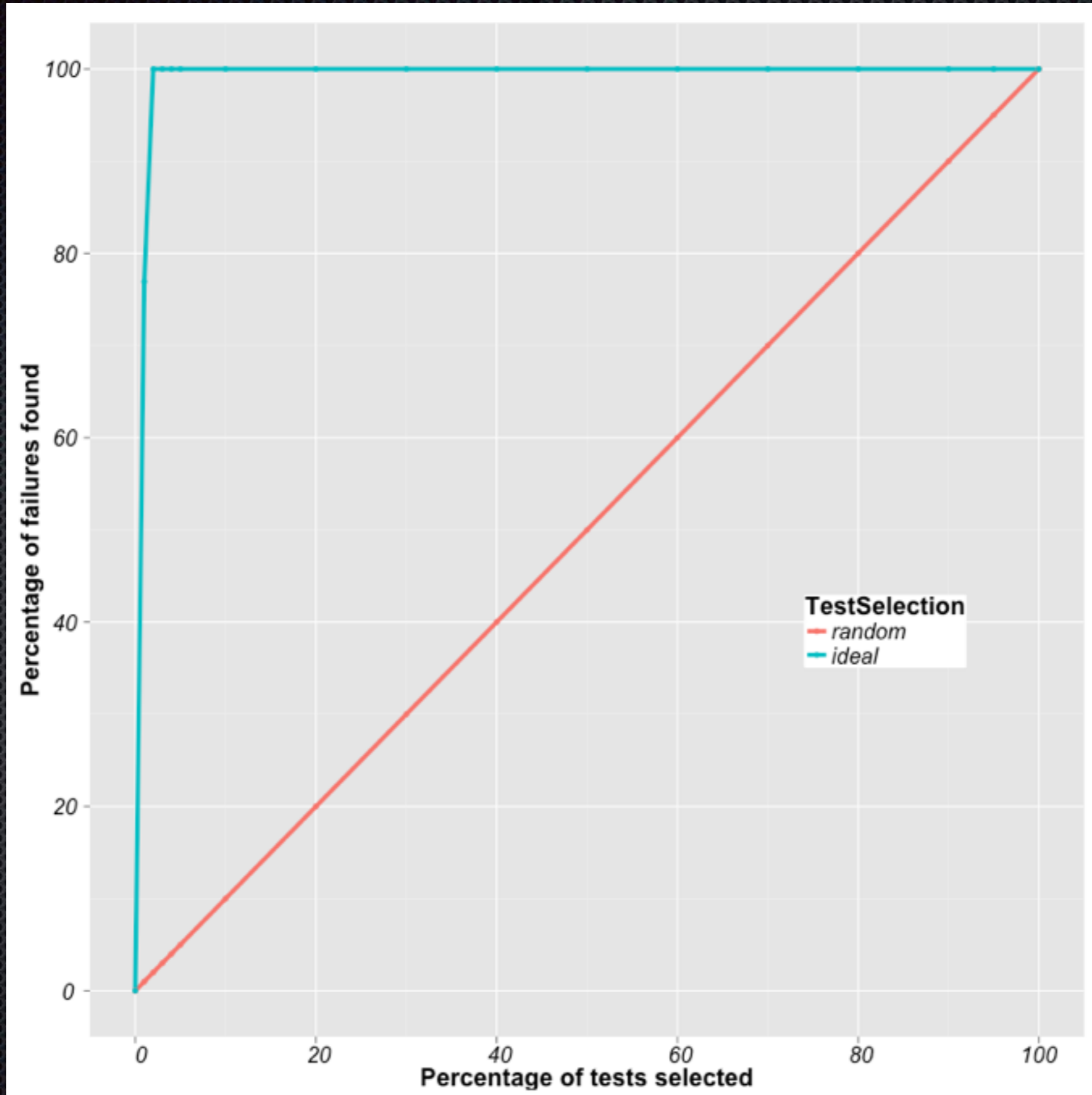
Faz not robust when selecting few test cases!
Large variation between param settings

How much can we gain?

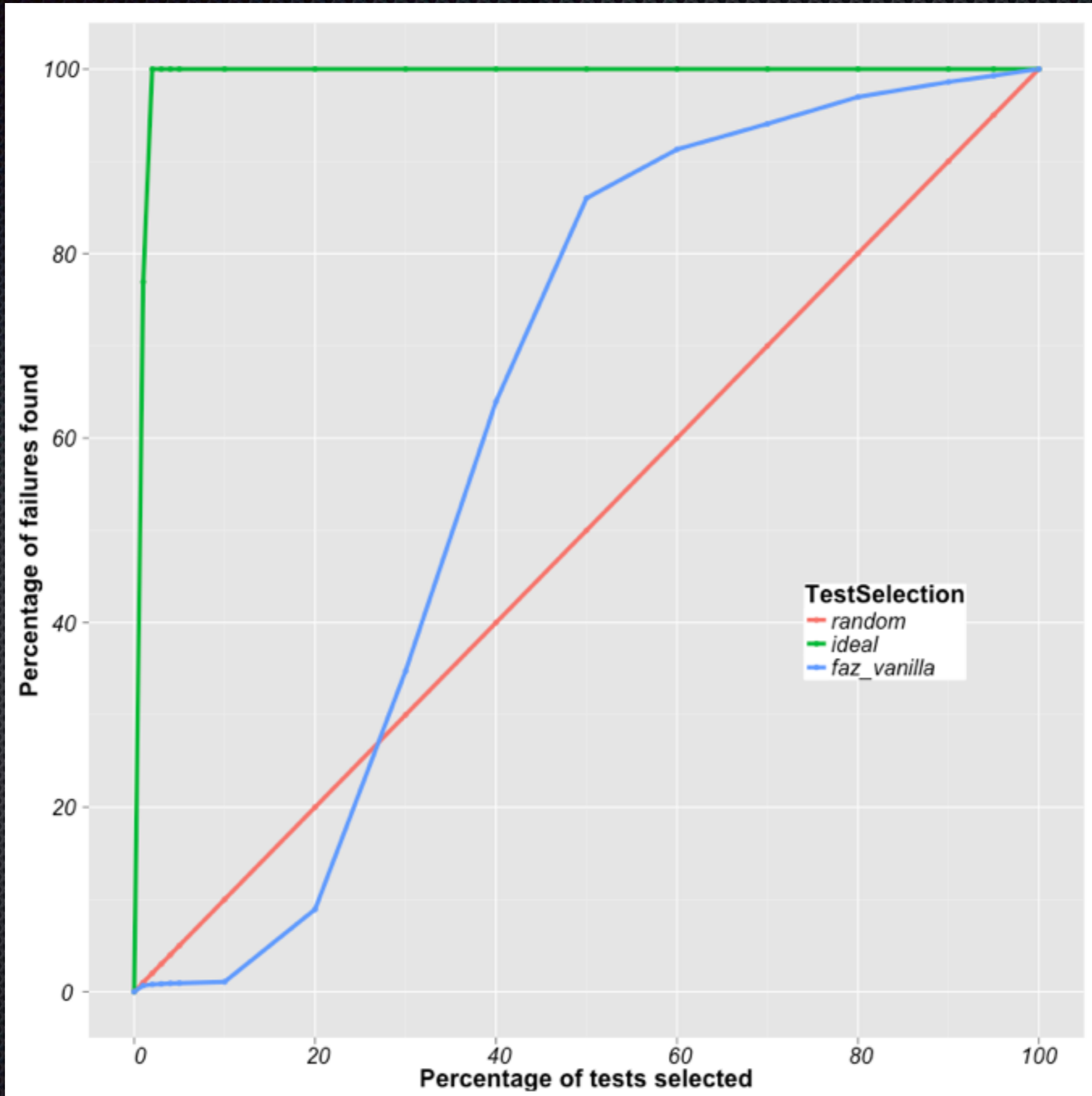
How much can we gain? (FDE-S curves)



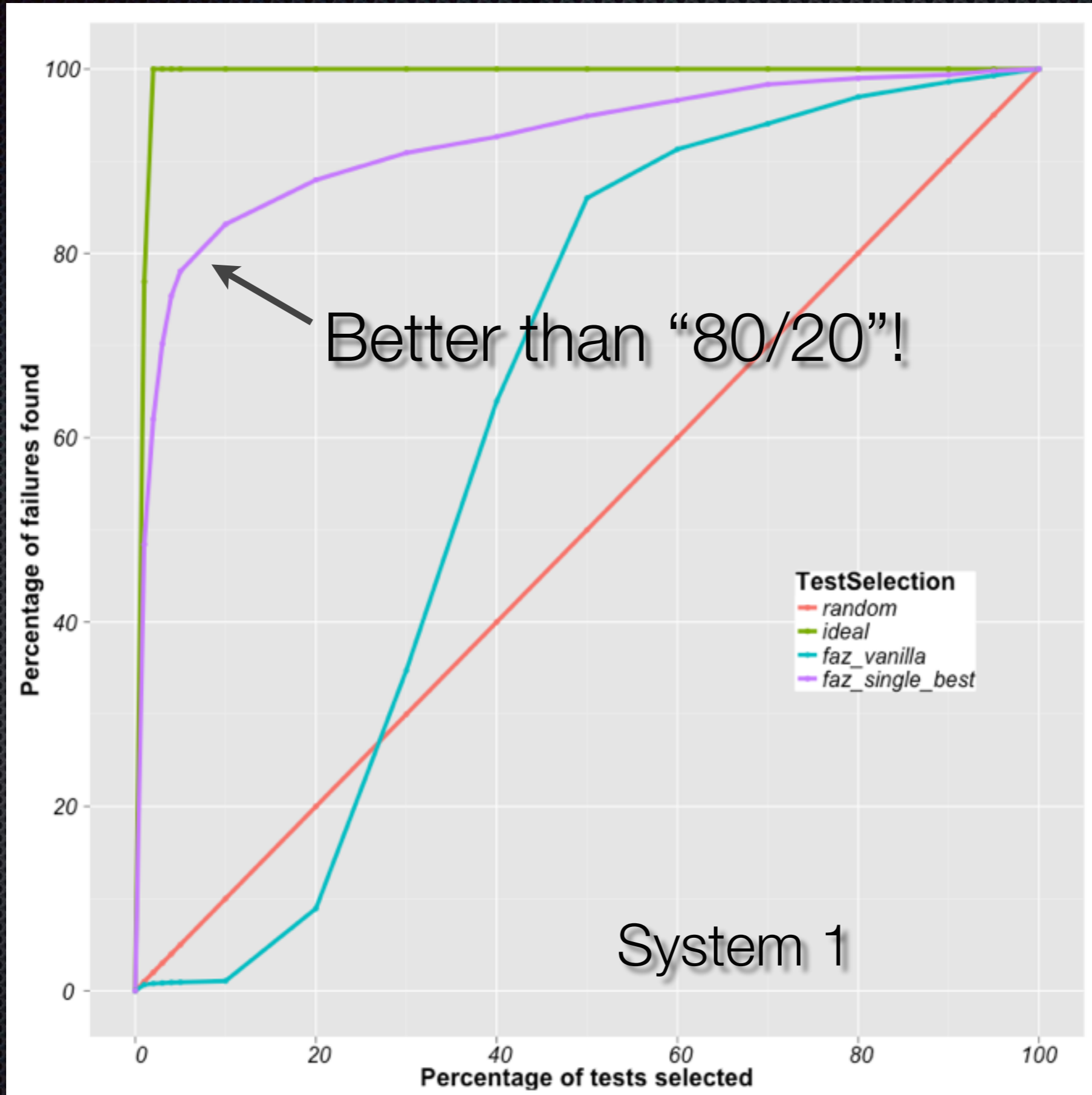
How much can we gain?



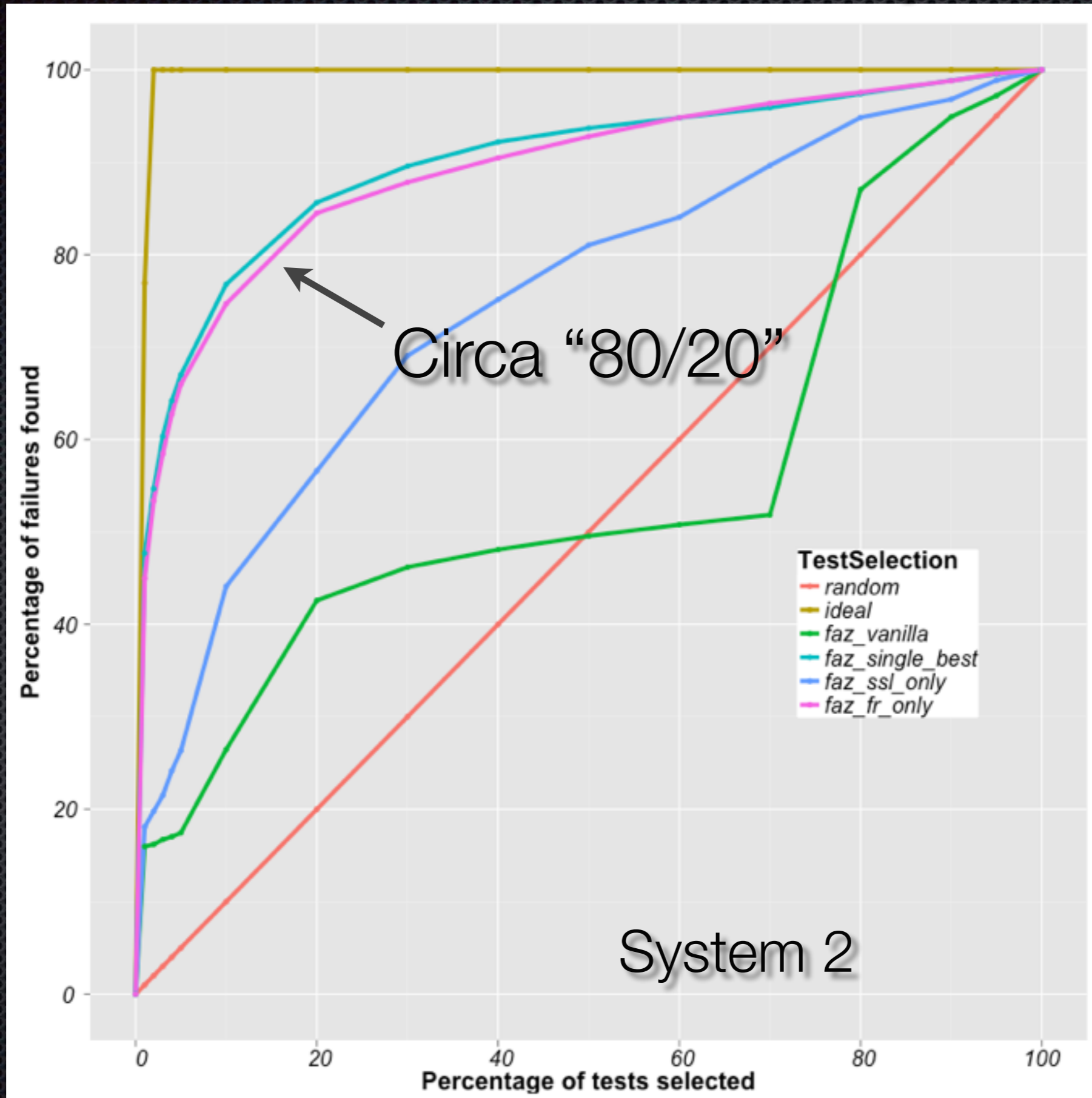
How much can we gain?



How much can we gain?



Are the result robust (for other system)?



Can we be sure this will work?

No, it will work if there are clear patterns in testing

For less regular systems we need more data.

The next natural step is to add info about source code changes

In [Wikstrand2009] we showed a simple file-based technique

Keeps a cache of when source code files were changed

This is one way we can extend the model if not good enough.

Combined method for test prioritization

If Top-10/20% selected, ~80-90% failures caught

“But want to be sure every test run every 4th test”

Add 20% more from prio list that not executed in last 4 tests

Ensures all tests executed at least every 4th full test but reduces total number of test runs -60%

Many different trade-offs/solutions possible
once optimization framework in place!

So what is the best way forward?

If someone forces me to make a recommendation today I would suggest:

- 1. Prioritize test cases on their historical failure rate (number of failures / number of executions)**
- 2. Refine the priorities based on source code file changes since last test run**
- 3. Add the time since last execution to prioritization**

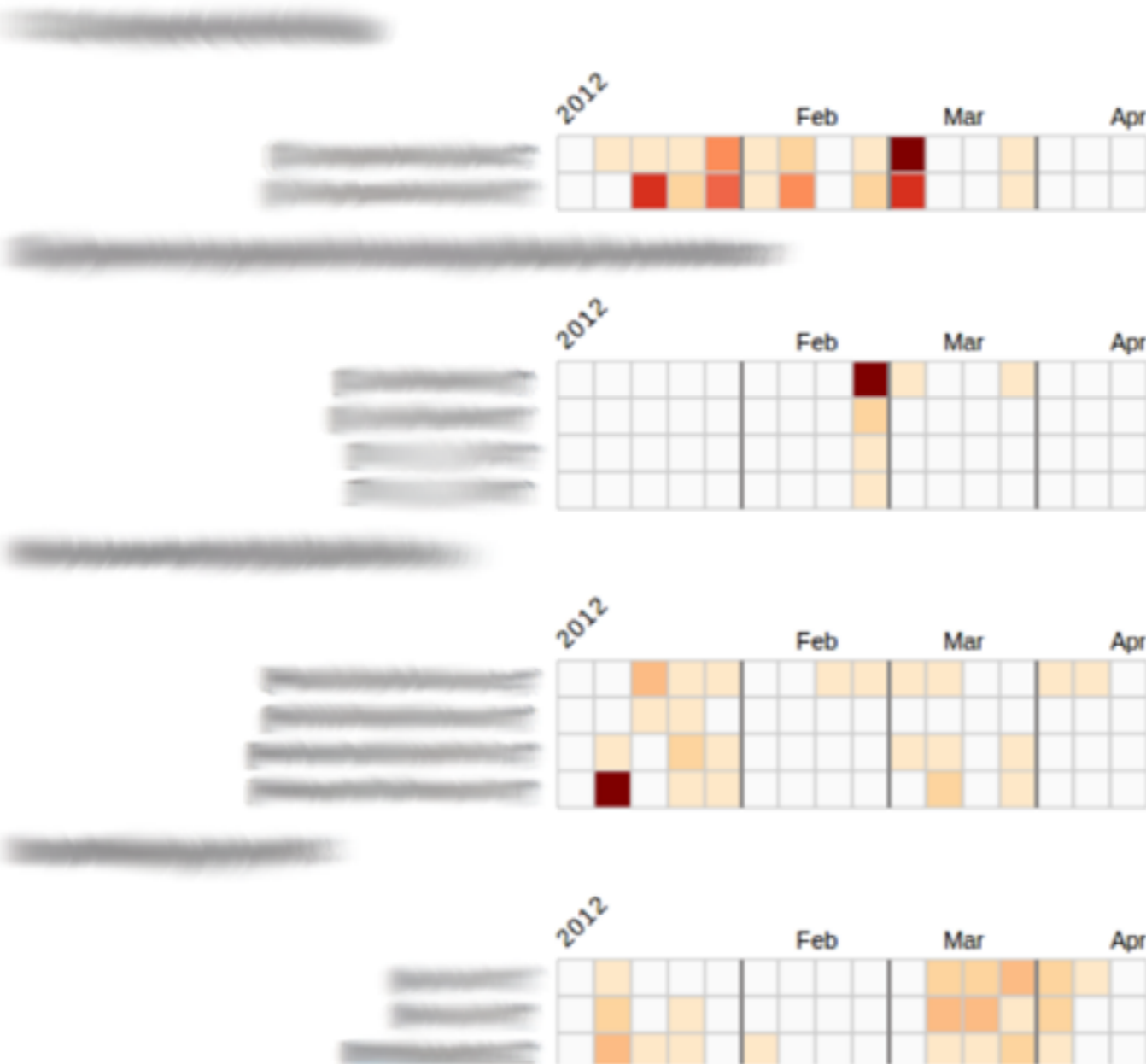
For all three “levels” use simple optimization to adapt to the project

Taking it Online

Flight code changes [about](#)

 Range from few to many line changes

* No matching module found



Test executions [about](#)

 Range from more failed to more successful

Row numbers: Total number of executions.



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Examples of real-world applications:

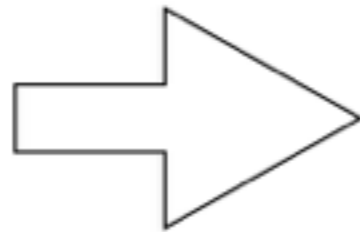
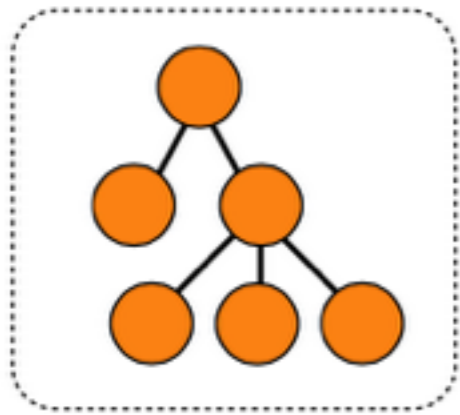
Optimizing test case selection

Generating complex test data

Searching for diverse test suites

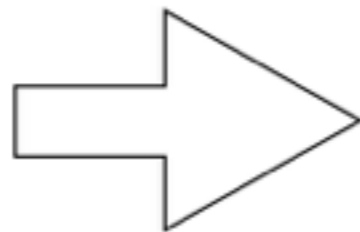
Highly-Structured Test Data

Some types of software take highly-structured inputs that must satisfy often complex constraints



data indexing systems

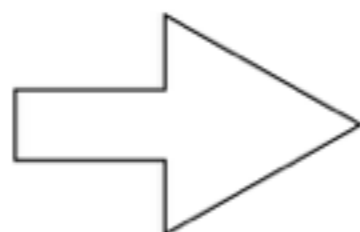
```
int main( ) {  
  printf("Hello");  
}
```



gcc

compilers

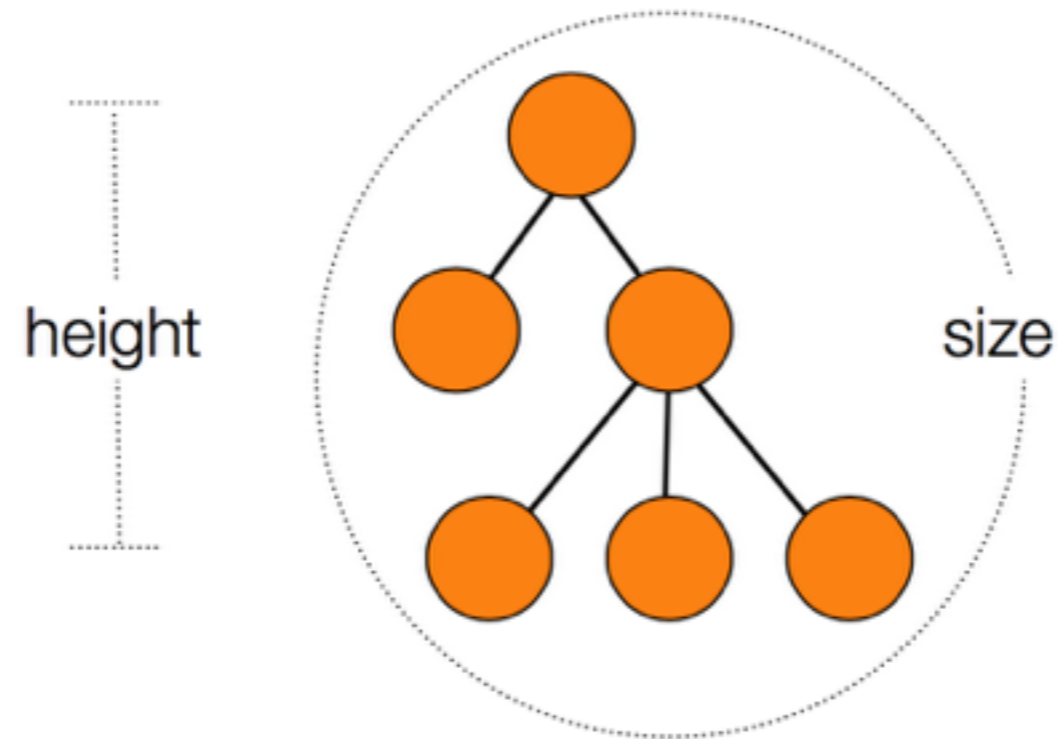
```
<html>  
<body>  
  <h1>A</h1>  
</body>  
</html>
```



HTML rendering engines

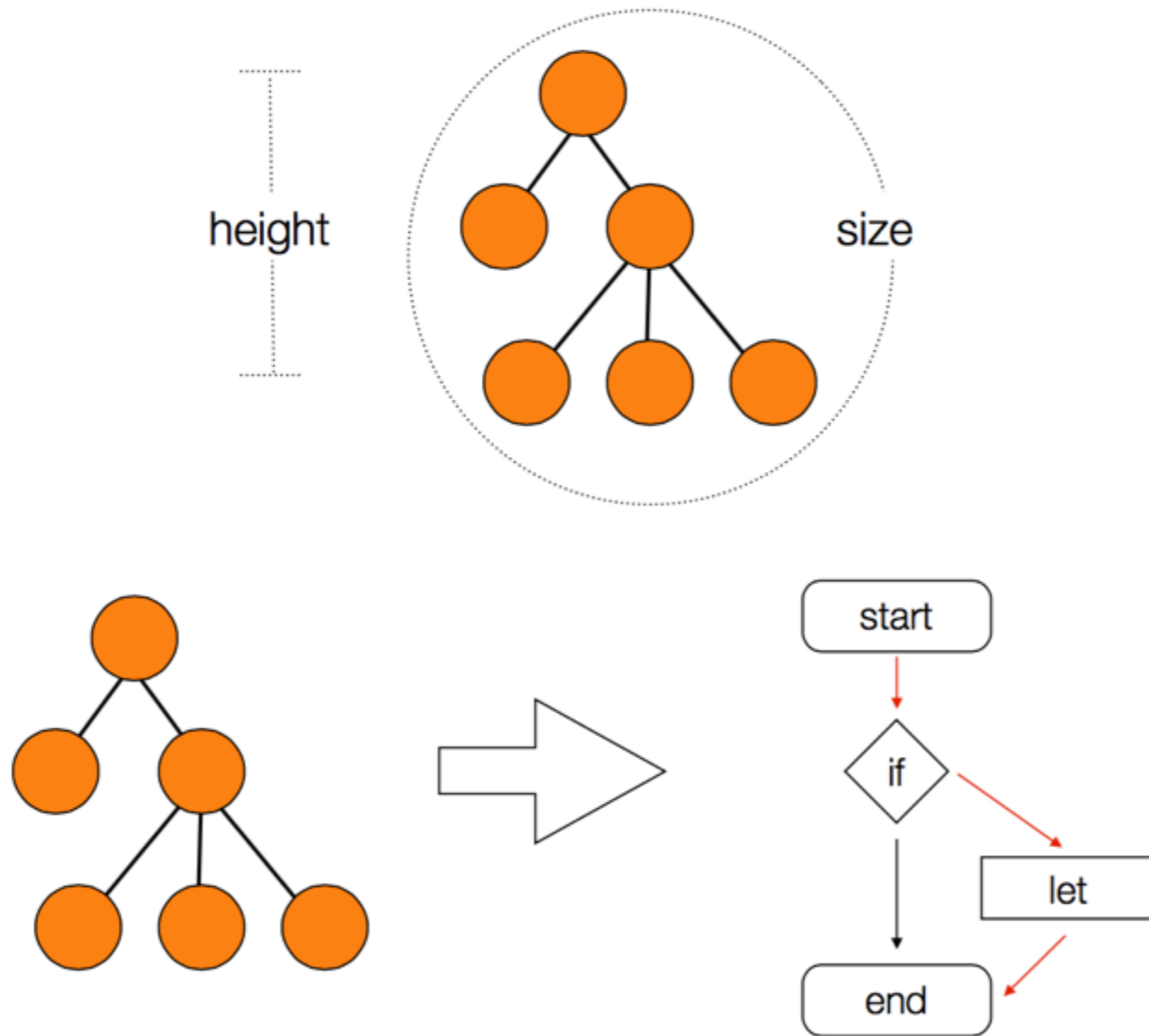
Bias Objectives

Effective testing may require the structured data to have specific intrinsic or extrinsic properties



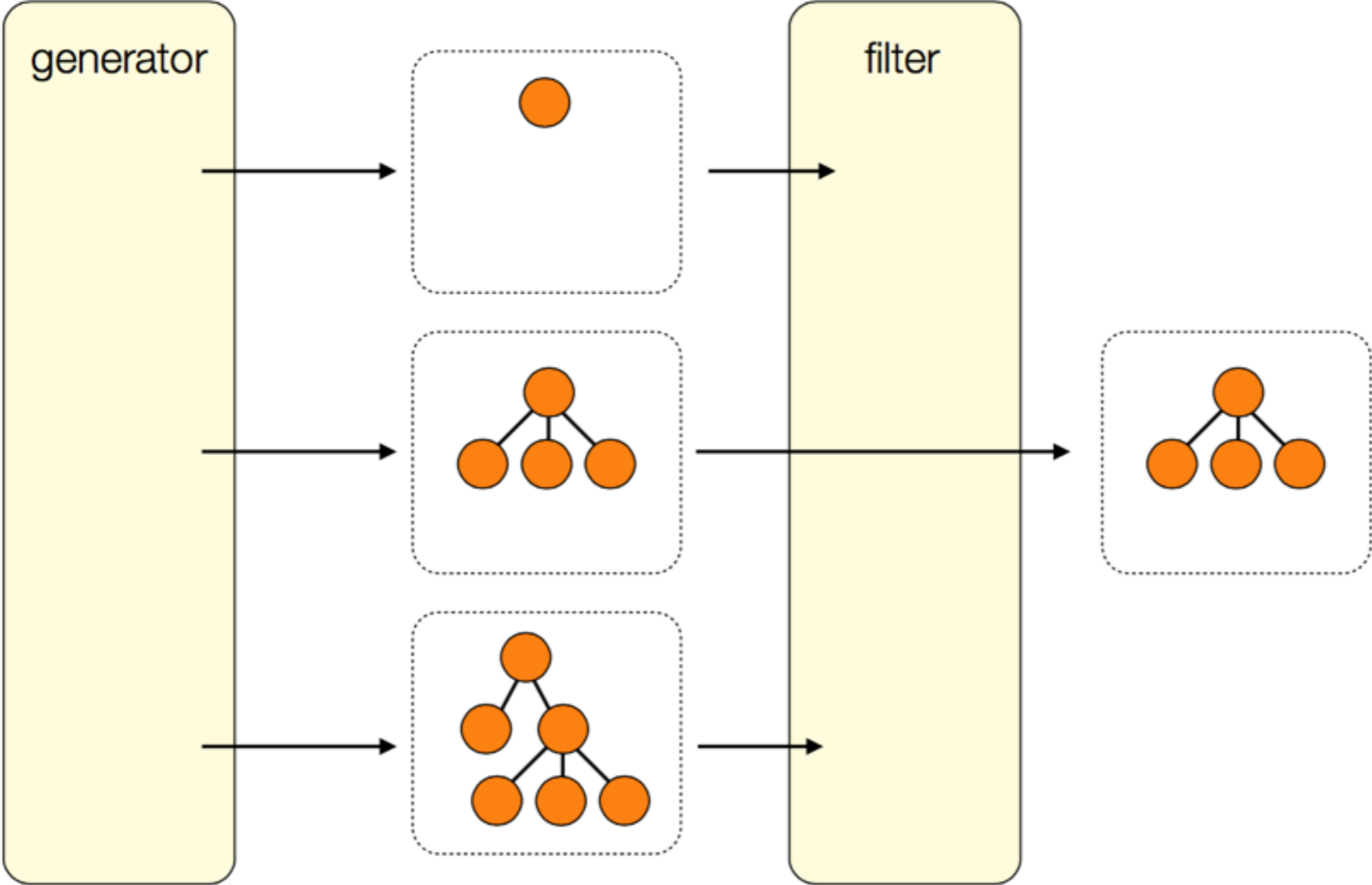
Bias Objectives

Effective testing may require the structured data to have specific intrinsic or extrinsic properties



Generation and Filtering

Bias objectives are typically met using a generator that builds test data with properties close to the desired values, often supplemented by an exact filter



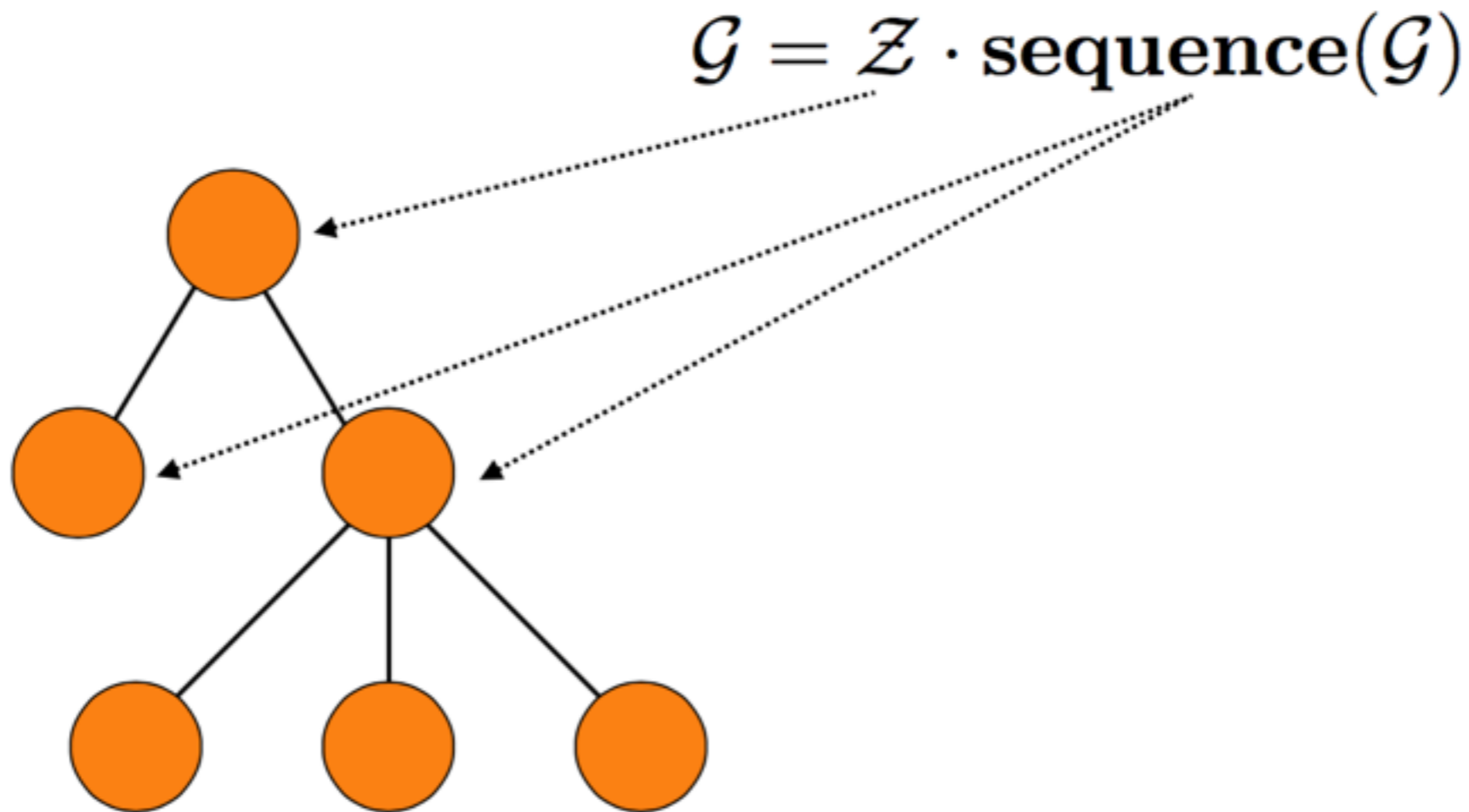
Boltzmann Samplers

A simple grammar describes how to generate test data using elementary operations that combine simpler objects

$$\mathcal{G} = \mathcal{Z} \cdot \mathbf{sequence}(\mathcal{G})$$

Boltzmann Samplers

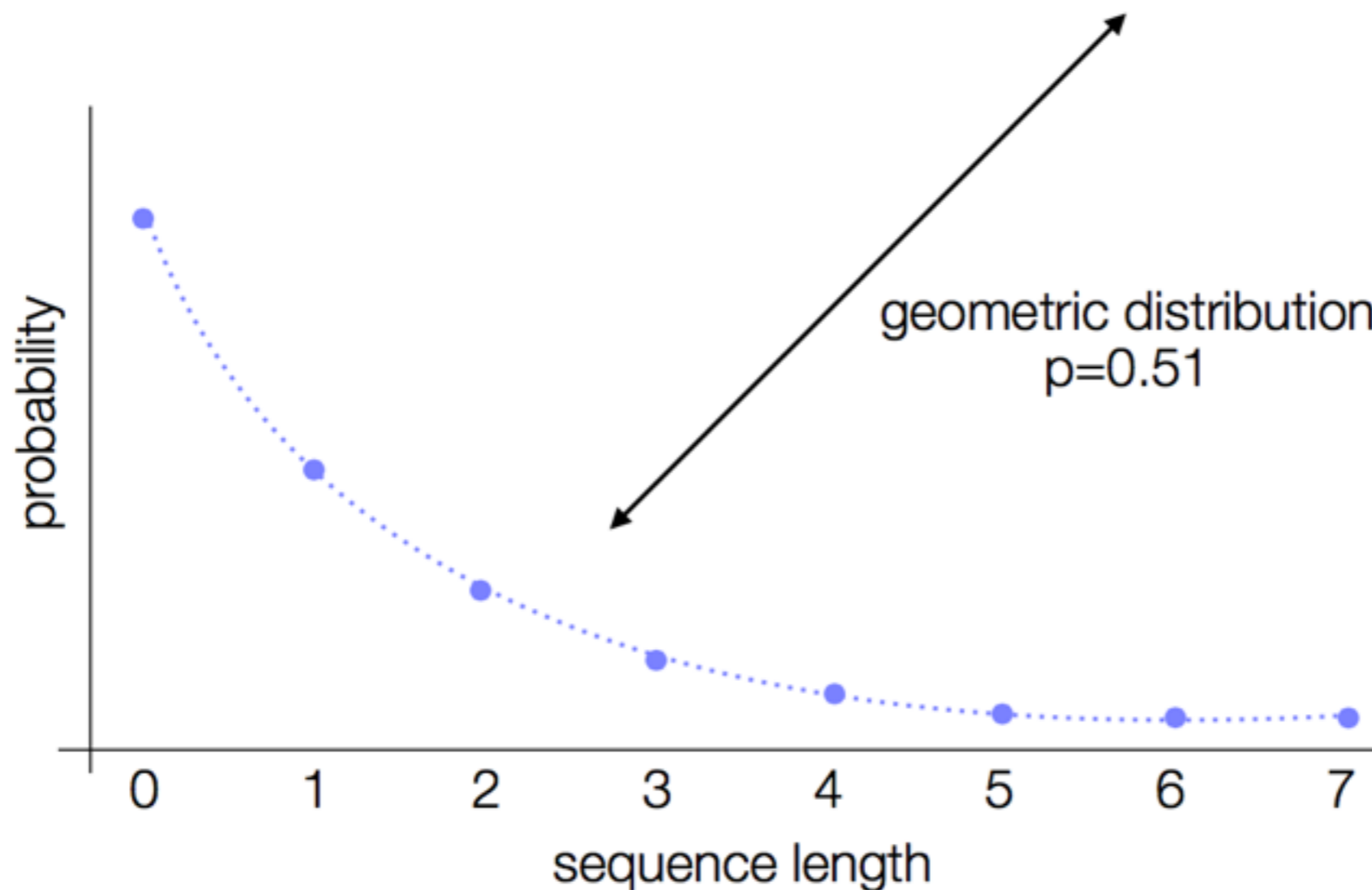
A simple grammar describes how to generate test data using elementary operations that combine simpler objects



Boltzmann Samplers

Mathematically tractable: it is relatively easy to calculate the local distributions at operators that produce a given mean tree size. But are limited to specific structures and only for the property of size.

$$\mathcal{G} = \mathcal{Z} \cdot \text{sequence}(\mathcal{G})$$



Stochastic Grammars

More flexible than Boltzmann samplers, but in general there is no analytical method for setting production weights to achieve bias objectives

$$S \rightarrow \text{GeneralTree}$$
$$\text{GeneralTree} \rightarrow \text{nodeLabel SubTrees}$$
$$\text{SubTrees} \rightarrow \epsilon \mid \text{GeneralTree SubTrees}$$

Non-Deterministic Programs

More flexible than grammars, but again there is generally no analytical methods of tuning non-determinism to achieve bias objectives

```
data Tree = Node Int [Tree] deriving (Eq, Show, Ord)

instance Arbitrary Tree where
  arbitrary = sized tree'
  where tree' n = liftM2 Node arbitrary (
    resize (n-1) (listOf' arbitrary))

listOf' gen = sized $ \n ->
  do k <- choose (0,n)
     if k == 0
       then vectorOf 0 gen
       else vectorOf k (resize ((n+k-1) `div` k) gen)
```

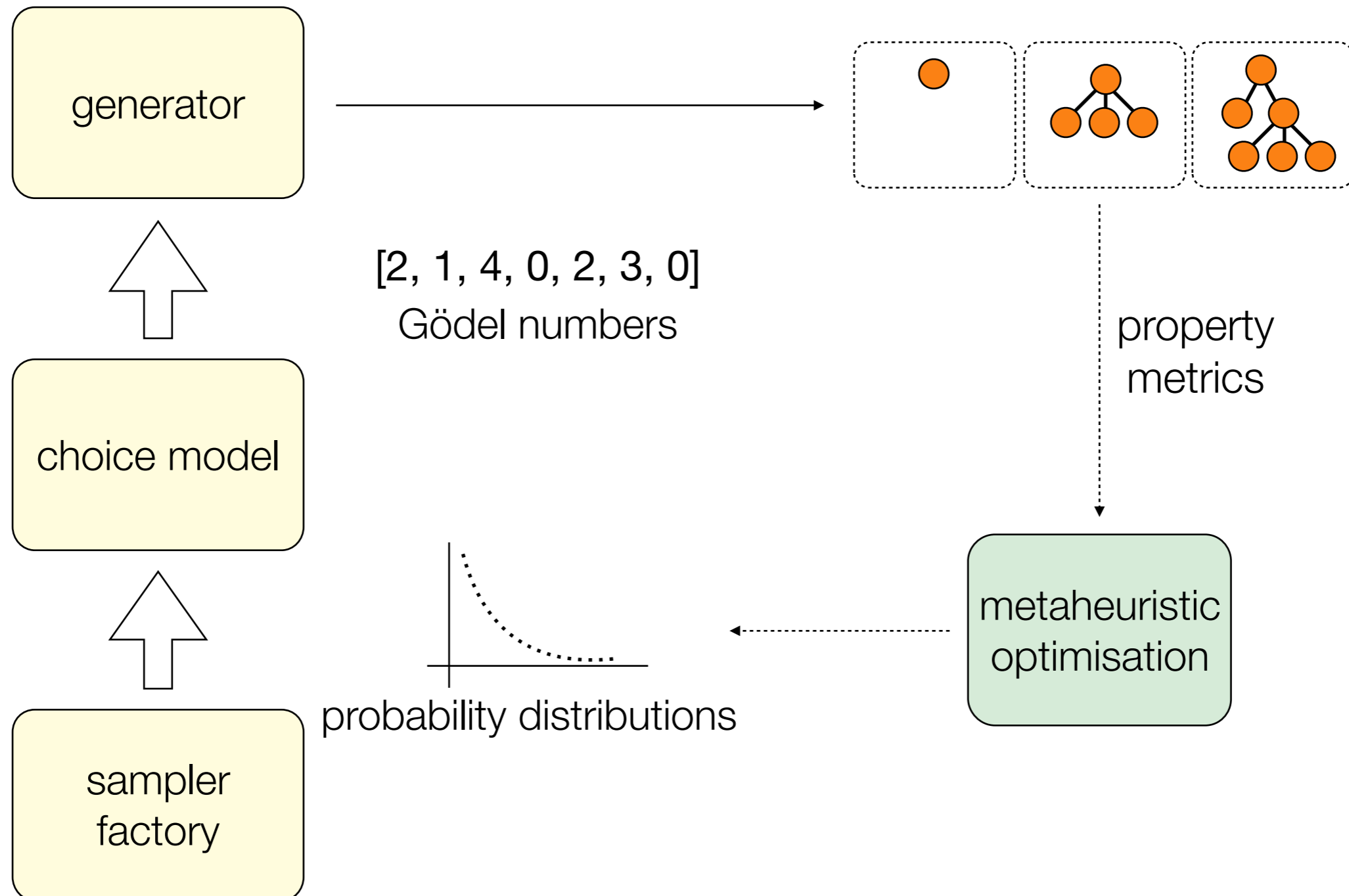
Objective

Require a technique:

- has the **flexibility** of non-deterministic programs
- can bias the generation to **any property** (not just size)
- can **automatically tune** the generator to achieve these biases

GödelTest Framework

Extracts a model of choice points from a non-deterministic generator; optimises the choice model using metaheuristic optimisation to meet bias objectives



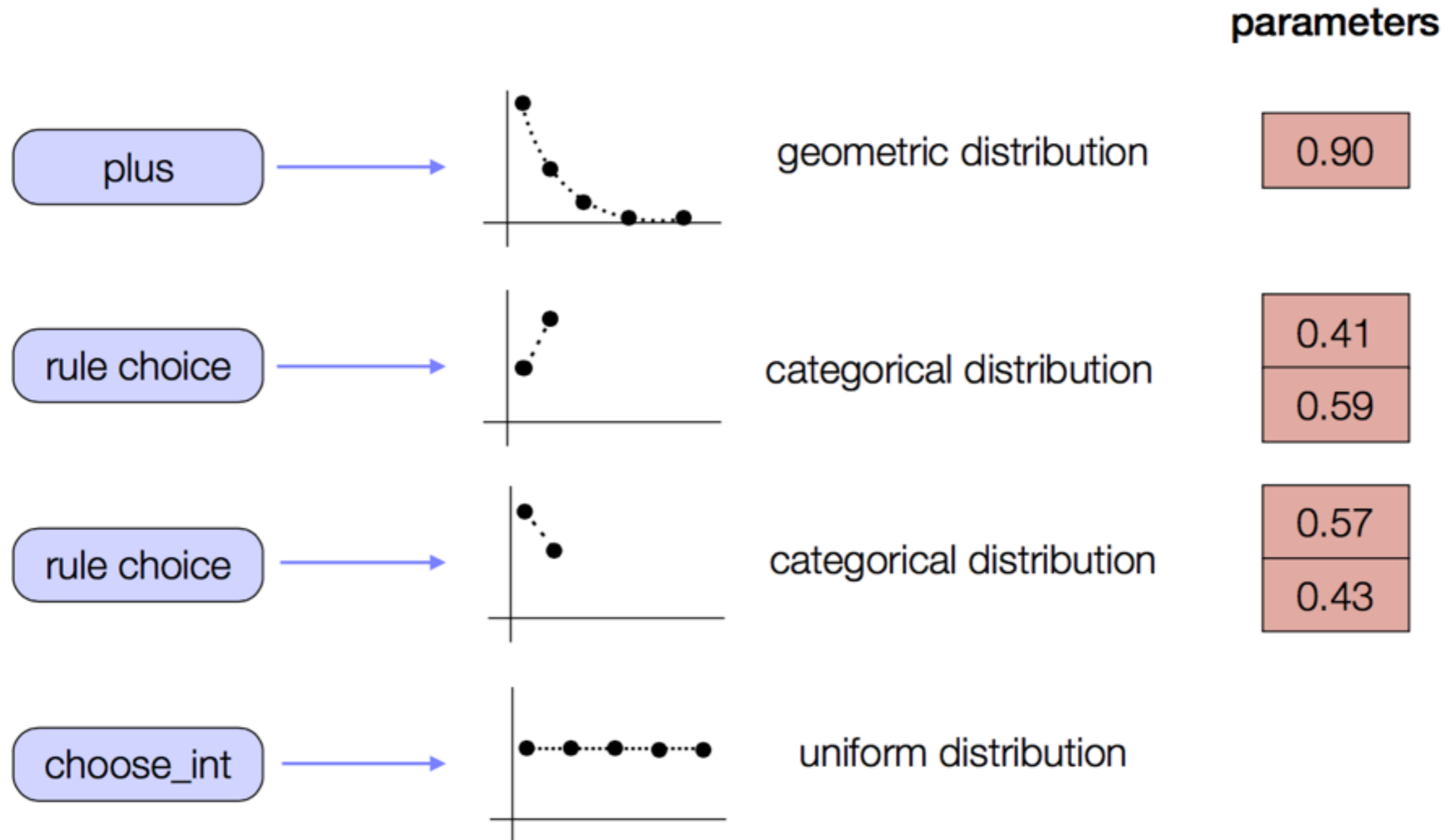
Generators

A DSL in Julia for constructing well-formed data;
non-determinism arises from a small set of implicit
and explicit choice points

```
# recursive generator for arithmetic expressions
@generator RecursiveExprGen begin
  start = expression
  expression = operand * " " * operator * " " * operand
  operand = number
  operand = "(" * expression * ")"
  number = (choose(Bool) ? "-" : "") * join(plus(digit))
  digit = string(choose(Int, 0, 9))
  operator = "+"
  operator = "-"
  operator = "/"
  operator = "*"
end
```

Sampler Factory

A sampler factory associates appropriate local distribution (samplers) to each choice point in the model



Metaheuristic Search

Metaheuristic search acts on the set of sampler parameters in order to modify the local distribution of Gödel numbers associated with each choice points

geometric distribution

0.90

categorical distribution

0.44

0.72

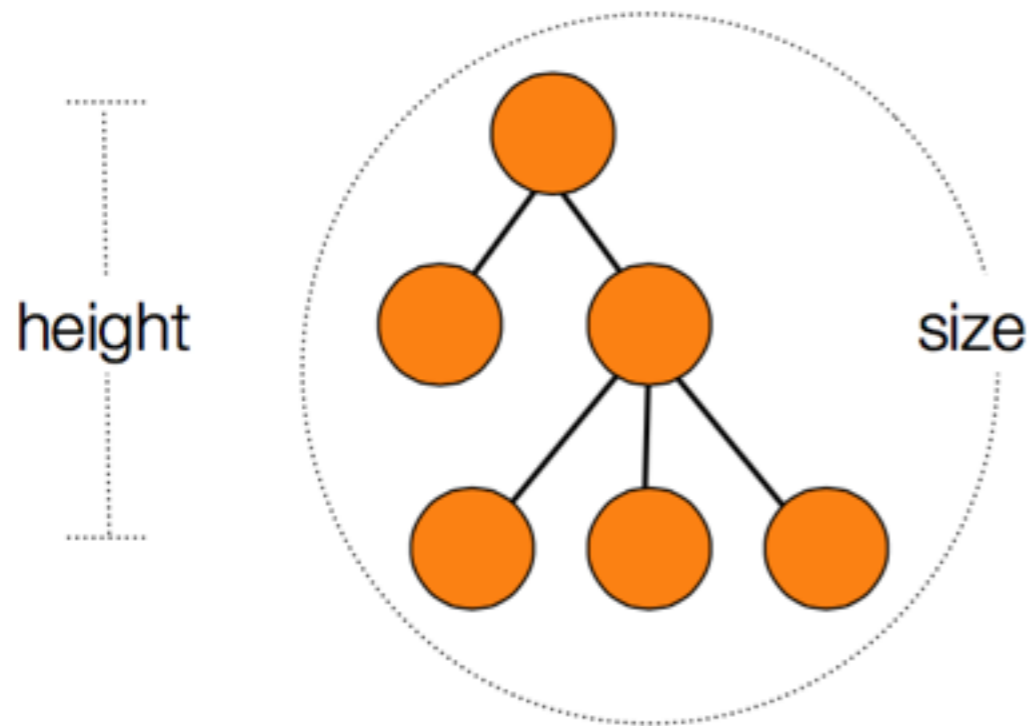
categorical distribution

0.77

0.53

Problem

Two target bias objectives specified in terms of tree size and height

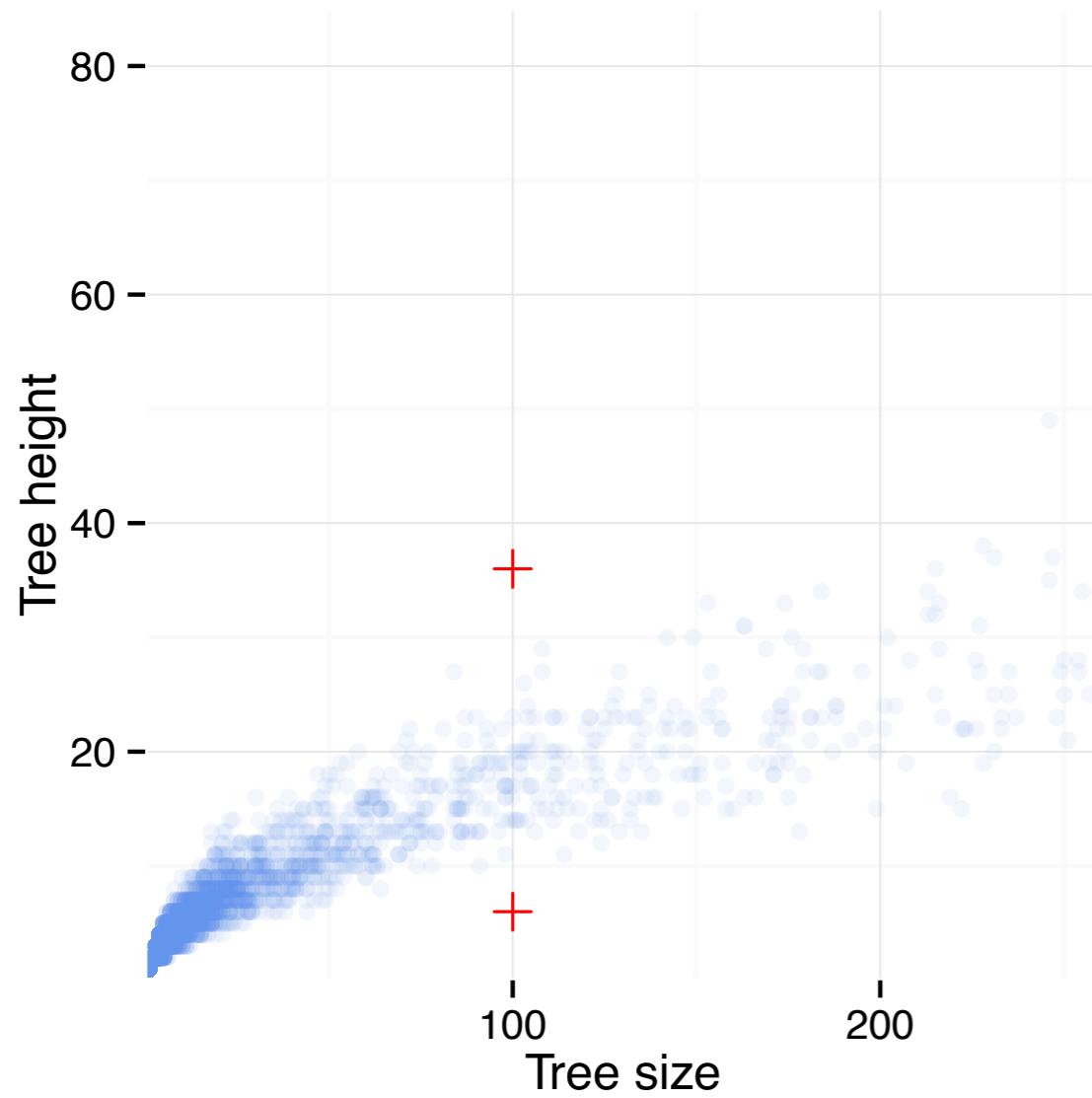


target 1: size = 100 AND height = 36

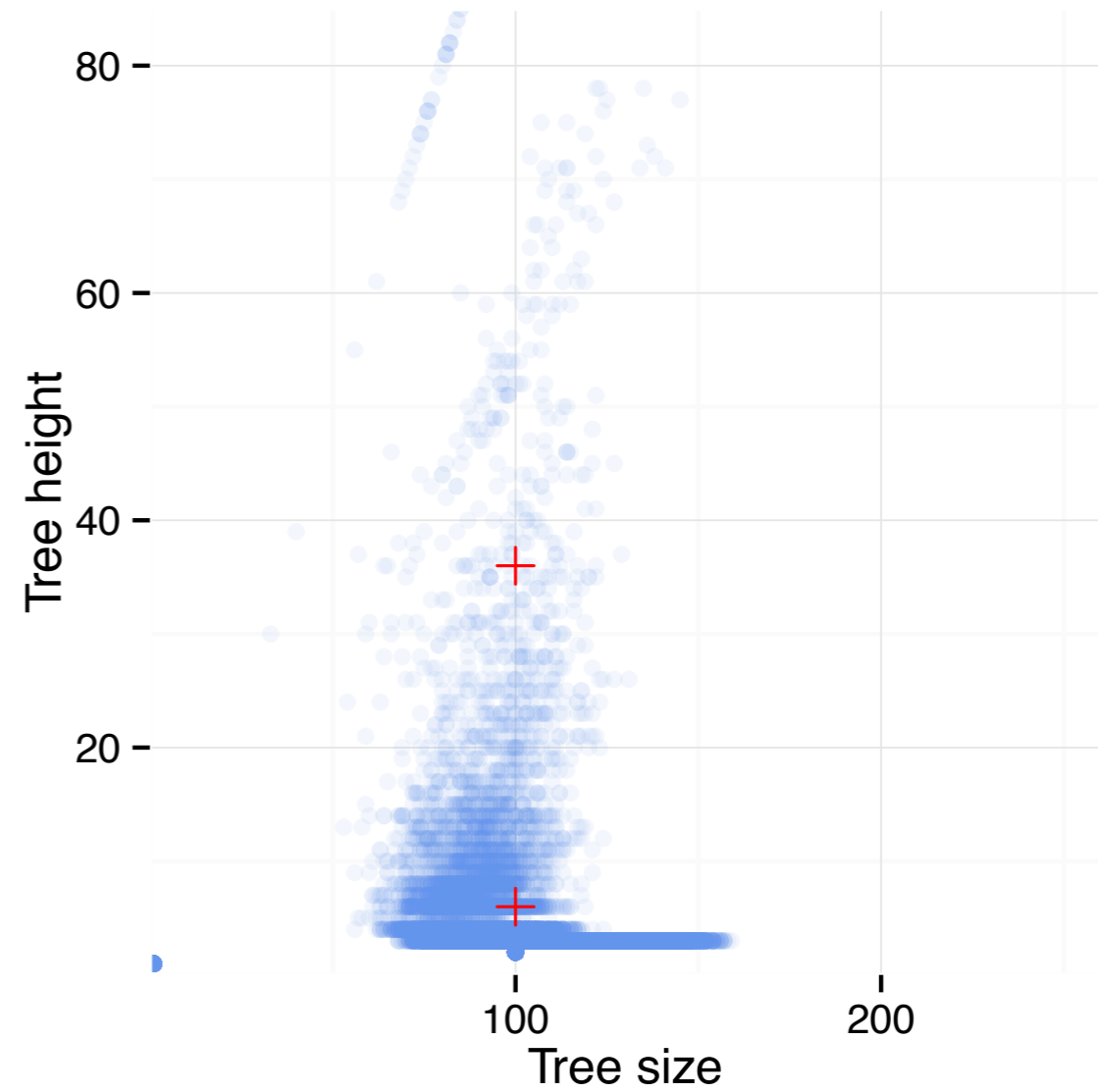
target 2: size = 100 AND height = 6

Results

Scatter plots show the distribution of tree sizes and heights; target bias objectives are indicated by crosses



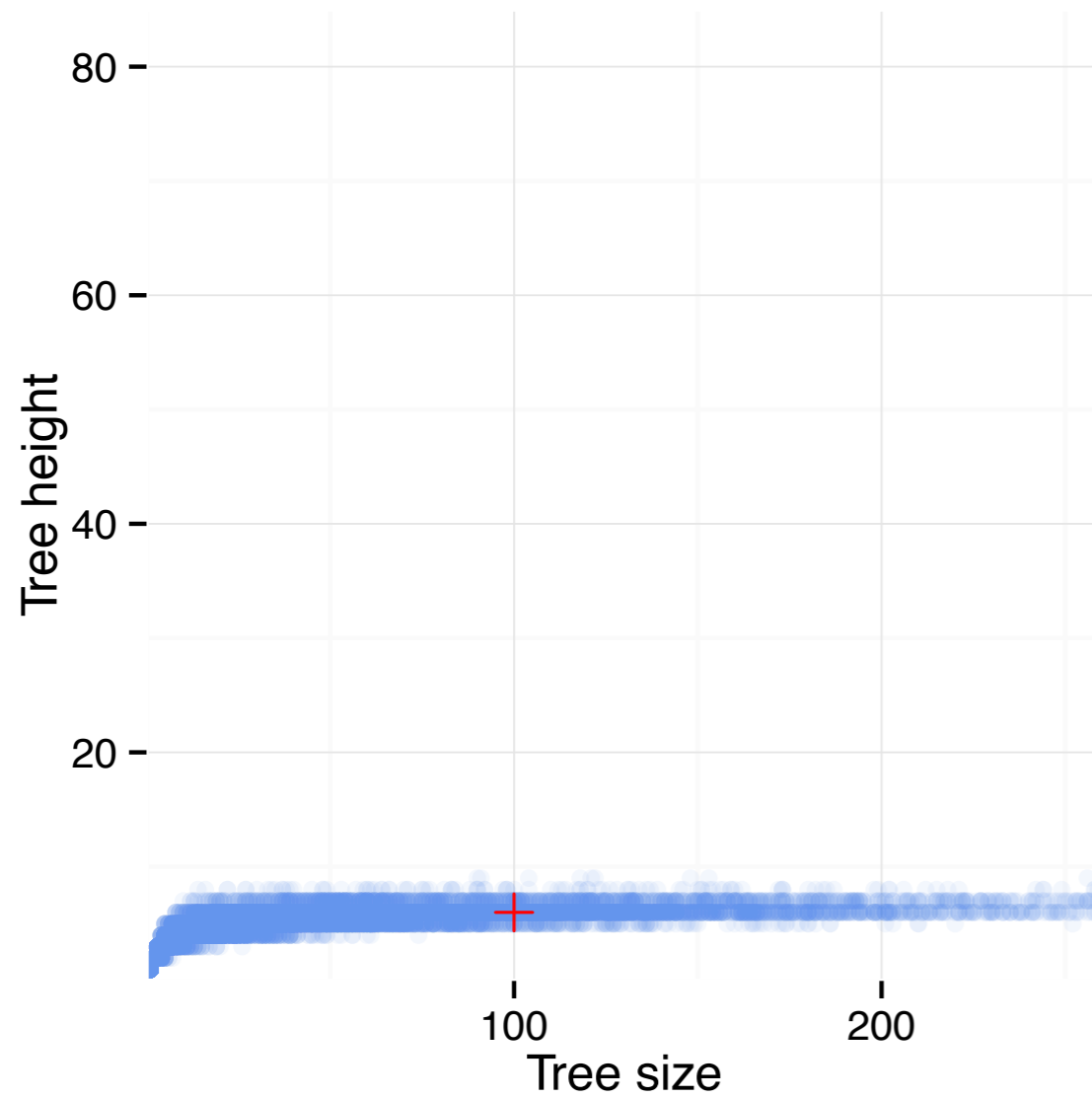
Boltzmann Sampler



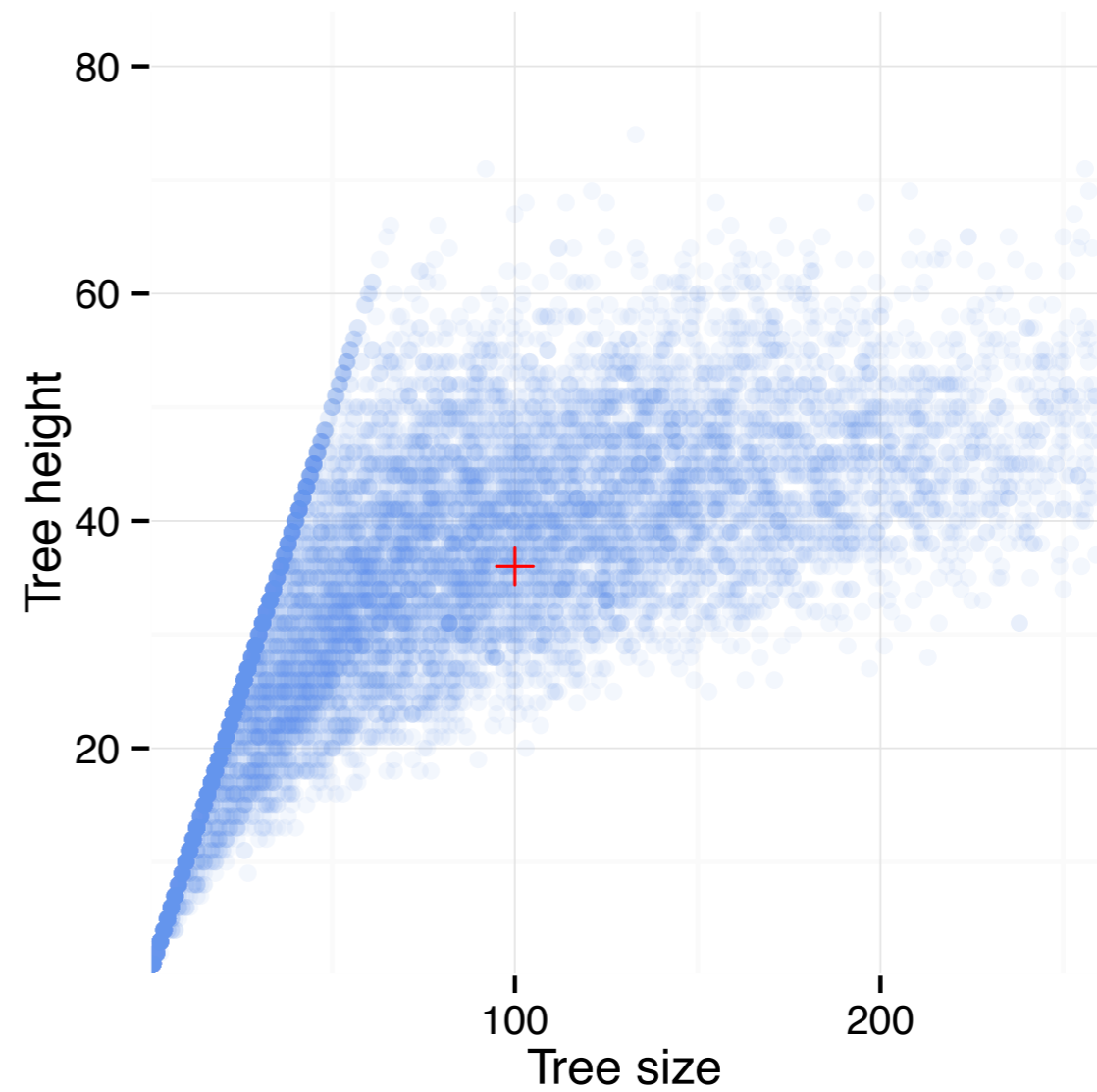
QuickCheck

Results

Scatter plots show the distribution of tree sizes and heights; target bias objectives are indicated by crosses



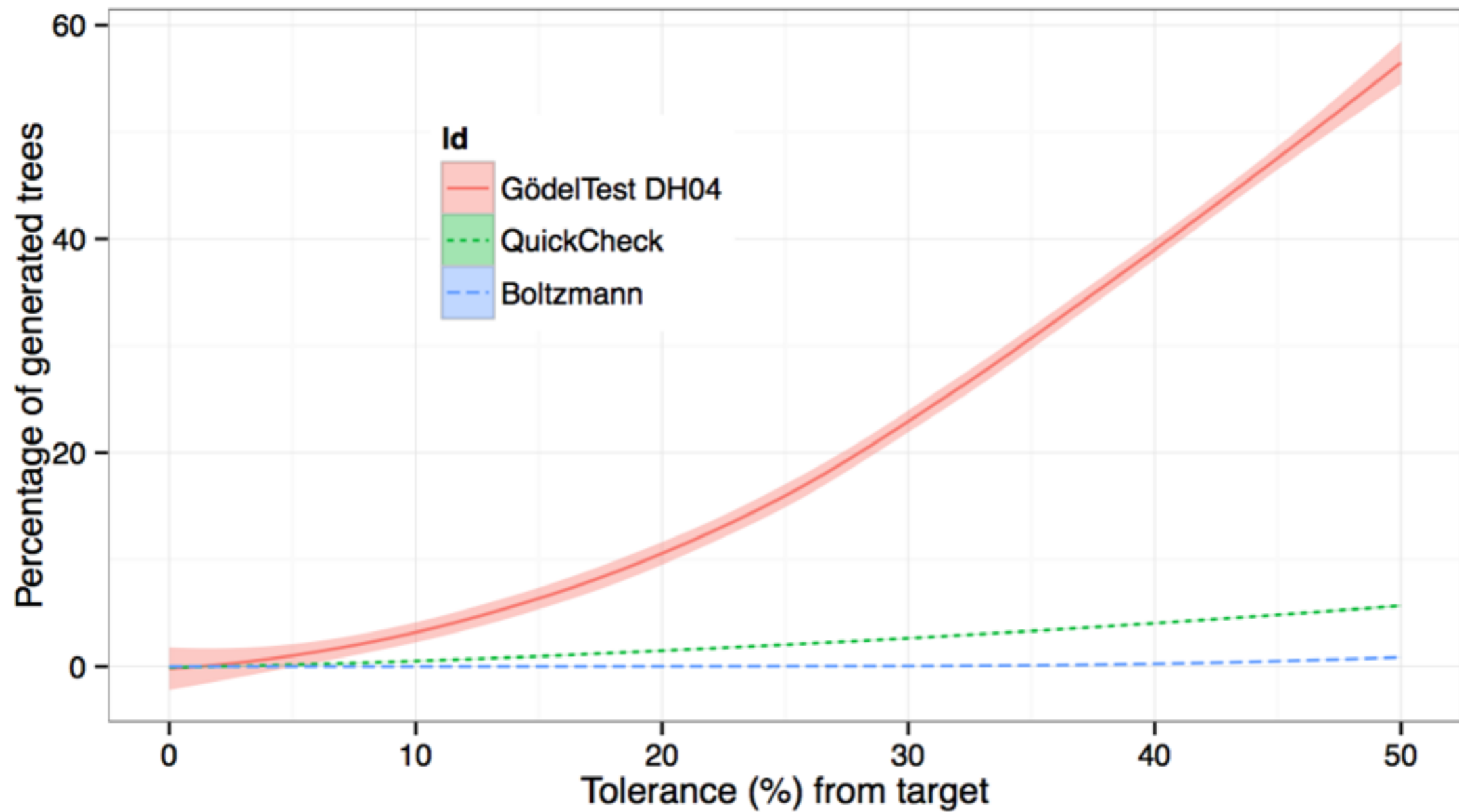
**GödelTest
(Decay Distribution)**



**GödelTest
(Decay Histogram)**

Results

The percentage of generated trees within a given tolerance of the bias objective target of size = 100, height = 36



Summary

GödelTest can **efficiently** generate **highly-structured** test data with specific **desirable properties**

The use of a non-deterministic program as a generator enables GödelTest to generate a wider range of data structures than Boltzmann samplers

The choice model is abstracted from the generator, and the local probability distributions associated with the model are optimised using metaheuristic search

Any property that is quantifiable can be used to specify a bias objective, and GödelTest is able to optimise for multiple bias objectives simultaneously

What is SBSE?

What is SBST?

Why is it not real-world (enough)?

Examples of real-world applications:

Optimizing test case selection

Generating complex test data

Searching for diverse test suites

Test Set Diameter: Quantifying the Diversity of Sets of Test Cases

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Abstract—A common and natural intuition among software testers is that test cases need to differ if a software system is to be tested properly and its quality ensured. Consequently, much research has gone into formulating distance measures for how test cases, their inputs and/or their outputs differ. However, common to these proposals is that they are data type specific and/or calculate the diversity only between pairs of test inputs, traces or outputs.

We propose a new metric to measure the diversity of sets of tests: the test set diameter (TSDm). It extends our earlier, pairwise test diversity metrics based on recent advances in information theory regarding the calculation of the normalized compression distance (NCD) for multisets. An advantage is that TSDm can be applied regardless of data type and on any test-related information, not only the test inputs. A downside is the increased computational time compared to competing approaches.

Our experiments on four different systems show that the test set diameter can help select test sets with higher structural and fault coverage than random selection even when only applied to test inputs. This can enable early test design and selection, prior to even having a software system to test, and complement other types of test automation and analysis. We argue that this quantification of test set diversity creates a number of opportunities to better understand software quality and provides practical ways to increase it.

has support in the research literature. For example, adaptive random testing [1] only adds a new, randomly-generated test case if it has large distance to existing test cases. But Chen et al. [1] also note that a number of testing methods such as Restricted Random Testing [2], Antirandom testing [3], and Quasi-Random Testing [4] are all based on the same idea: ‘evenly spreading’ test cases over the input domain. Critical to the success of these techniques is a generically applicable diversity measure and Chen et al. go as far as saying that ‘We have come to realise that “even spreading” can be better described as a form of diversity’ [1]. They describe a distance calculation scheme based on the category-partition (partition testing) method, but it requires that the tester manually identifies categories and levels which can be varied.

Most approaches to quantifying diversity among test cases are specific to a certain type of data. It is common to assume that the data is numeric since there are a multitude of existing distance functions that can then be applied. One example is the approach of Bueno et al. [5] which selects test sets that maximize the sum of the distances from each test input to its nearest neighbor. In their empirical work they use the Euclidean distance which requires the inputs to be numerical vectors. More recently, Alshawan et al. [6] proposed to select

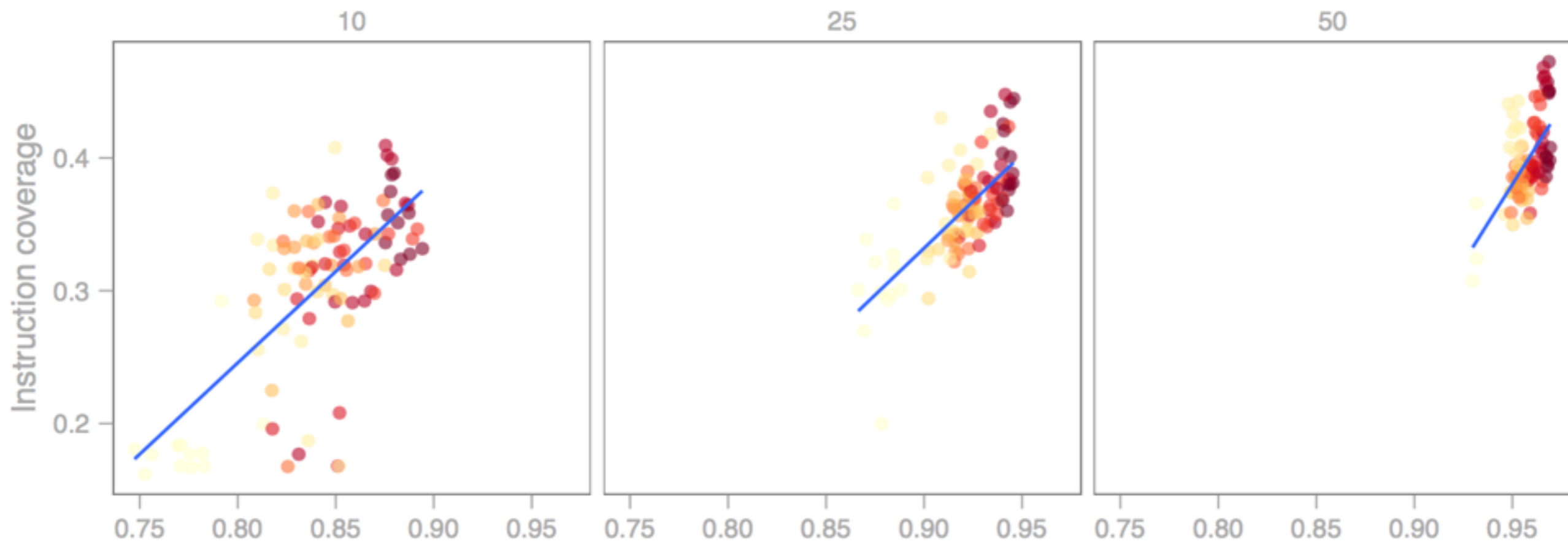
The Kolmogorov complexity of a string of symbols, x , is the length of the shortest program that outputs x [10]. It is a measure of the information contained in x , and we denote it here as $K(x)$. The conditional Kolmogorov complexity of x given y , denoted $K(x|y)$ is the length of the shortest program that outputs x given the input y .

Normalized Info Distance

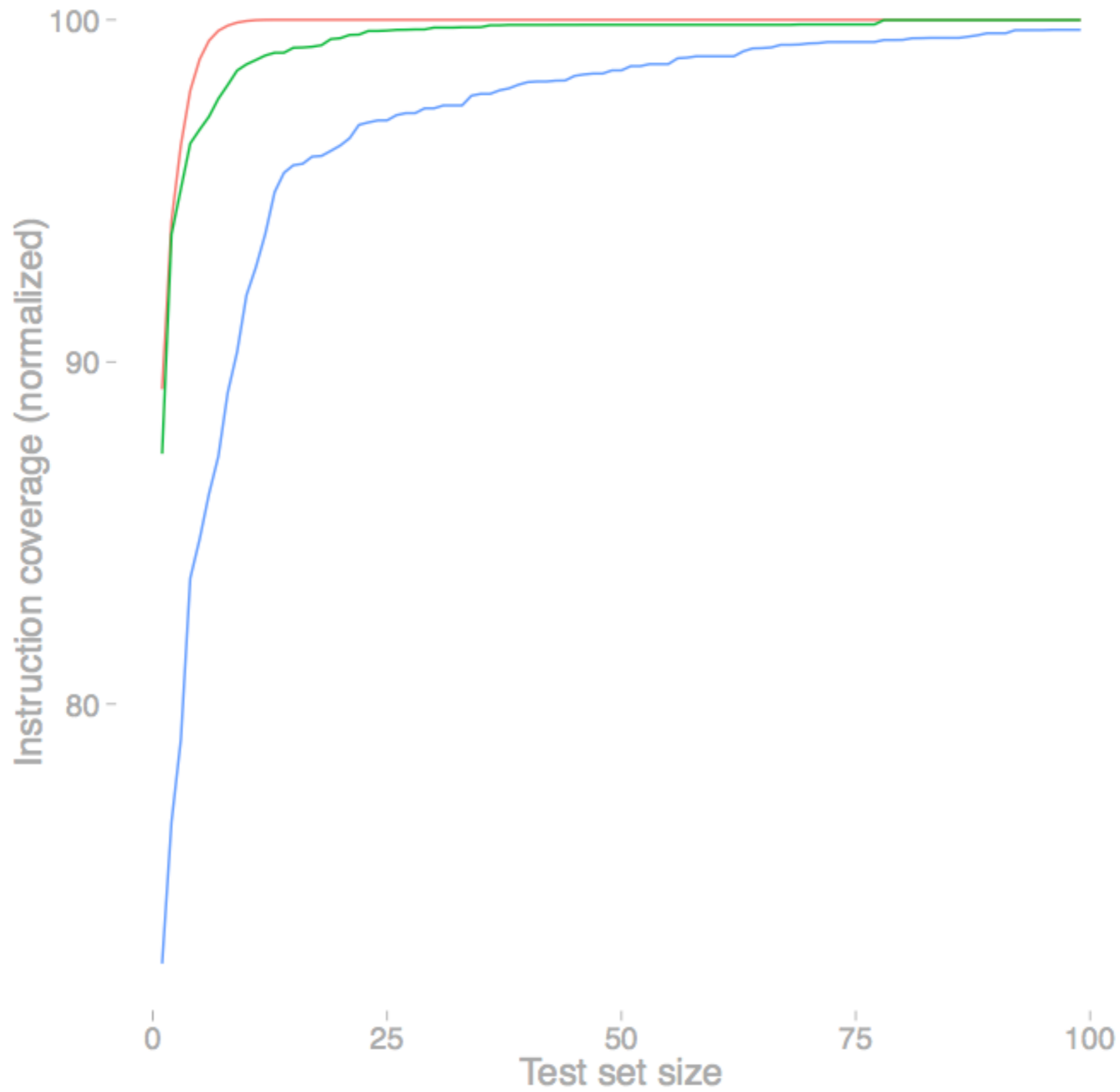
$$\text{NID}(x, y) = \frac{\max\{K(x|y), K(y|x)\}}{\max\{K(x), K(y)\}}$$

Normalized Compression Distance

$$\text{NCD}(x, y) = \frac{C(xy) - \min\{C(x), C(y)\}}{\max\{C(x), C(y)\}}$$



(a) JEuclid



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My other colleagues, students and co-authors for the research presented here...

Thank you! Questions?

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