Fourth Halmstad Summer School on Testing
June 9-12, 2014

Testing and Verification in ACL2

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June 12, 11:00 – 12:30

Digital circuit design verification

Commercial success for theorem provers
- ✓ AMD, Centaur Tech: ACL2
- ✓ Hewlett-Packard (in the engineering days): Isabelle
- ✓ Intel: Forte (model checking, lightweight HOL)

Digital circuits have specs
- ✓ facilitates use of formal methods
- ✓ software bug or feature?

Circuit verification
- ✓ VLSI design (eg, VHDL) – testing and fabrication
- ✓ formal model (eg, ACL2) – testing and verification
- ✓ design ≡ model ?

Small example: ripple-carry adder
- ✓ to illustrate the general idea

Binary numerals

✓ Conventional rendering

$x_n x_{n-1} \ldots x_2 x_1 x_0$
where each x_k is a binary digit (0 or 1)
x_n is high-order bit, x_0 is low-order bit

✓ Formal representation for our models

$[x_0\ x_1\ x_2\ \ldots\ x_n]$
bit-sequence in reverse order: low-order bit first, high-order last

✓ Converting between numerals and numbers

Definitional properties

(bits 0) = nil	{bits0}
(bits (n+1) = (cons (mod (n+1) 2) (bits(floor (n+1) 2)))	{bits1}
(numb nil) = 0	{nmb0}
(numb(cons x xs)) = x + 2∗(numb xs)	{nmb1}

Derivable property: numb inverts bits

(numb(bits n)) = n when n is a non-negative integer {bits-id}

Half-adder circuit and formal model

numerals in
numerals out

+	0	1	x
0	00	01	
1	01	10	
y			

carry bit of x+y / sum bit of x+y

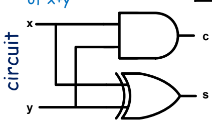
carry bit truth table

xy	c
00	0
01	0
10	0
11	1

sum bit truth table

xy	s
00	0
01	1
10	1
11	0

half-adder circuit



half-adder model

```
(defun and-gate (x y)
  (if (and (= x 1) (= y 1)) 1 0))
(defun or-gate (x y)
  (if (or (= x 1) (= y 1)) 1 0))
(defun xor-gate (x y)
  (if (and (= x 1) (= y 1))
      0
      (or-gate x y)))
(defun half-adder (x y)
  (list (xor-gate x y) (and-gate x y)))
```

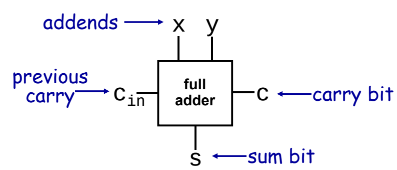
s 2-bit numeral c

correctness property
((numb(halfadder x y)) = (x + y))

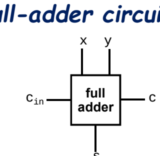
Adding binary numerals

```
 11011101   ← carry
  01011101
+ 11010101
  00110010
```

Half-adder is not enough
Need 3 inputs at each stage
- ✓ 2 bits from addends
- ✓ carry bit from previous position



Full-adder circuit

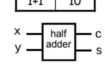


full-adder

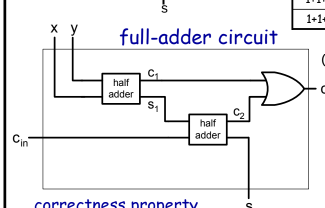
c_{in}+x+y	cs
0+0+0	00
0+0+1	01
0+1+0	01
0+1+1	10
1+0+0	01
1+0+1	10
1+1+0	10
1+1+1	11

half-adder

x+y	cs
0+0	00
0+1	01
1+0	01
1+1	10



full-adder circuit



full-adder model

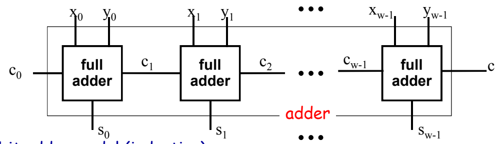
```
(defun full-adder (c-in x y)
  (let* ((h1 (half-adder x y))
         (s1 (first h1))
         (c1 (second h1))
         (h2 (half-adder s1 c-in))
         (s  (first h2))
         (c2 (second h2))
         (c  (or-gate c1 c2)))
    (list s c)))
```

in: 3 bits
out: 2-bit numeral

correctness property
(numb(fullAdder c_{in} x y) = (c_{in} + x + y))

w-bit ripple-carry adder

(adder c_0 [x_0 x_1 … x_{w-1}] [y_0 y_1 … y_{w-1}]) = [[s_0 s_1 … s_{w-1}] c]



w-bit adder model (inductive)

```
(defun adder (c0 x y)  ; in: carry-bit and two w-bit numerals
  (if (consp x)
      (let* ((x0 (first x)) (xs (rest x)) (y0 (first y)) (ys (rest y))
             (a0 (full-adder c0 x0 y0)) (s0 (first a0)) (c1 (second a0))
             (a  (adder c1 xs ys)) (ss (first a)) (c (second a)))
        (list (cons s0 ss) c)) ; {add1} ; out: w-bit numeral and carry
      (list nil c0)))          ; {add0}
```

correctness property

(numb(append [s_0 s_1 … s_{w-1}] [c]))
= (numb[x_0 x_1 … x_{w-1}]) + (numb[y_0 y_1 … y_{w-1}]) + c_0
where [[s_0 s_1 … s_{w-1}] c] = (adder c_0 [x_0 x_1 … x_{w-1}] [y_0 y_1 … y_{w-1}])

demo 6

Bignum adder numerals of unbounded length

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Simple problem to start: increment by 1

```
; (add-1 x) = numeral for (+ 1 (numb x)))
```


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```
; (add-1 x) = numeral for (+ 1 (numb x)))
; (add-1 nil) = ??                              {add1nil}
```


Bignum adder numerals of unbounded length

Simple problem to start: increment by 1

```
; (add-1 x) = numeral for (+ 1 (numb x)))
; (add-1 nil) = (list 1)                        {add1nil}
```


Bignum adder numerals of unbounded length

Simple problem to start: increment by 1

```
; (add-1 x) = numeral for (+ 1 (numb x)))
; (add-1 nil) = (list 1)                        {add1nil}
; (add-1 (cons 0 x)) = ??                        {add10}
```

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; (add-1 x) = numeral for (+ 1 (numb x)))
; (add-1 nil) = (list 1)                        {add1nil}
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; (add-1 nil) = (list 1)                        {add1nil}
; (add-1 (cons 0 x)) = (cons 1 x)                {add10}
; (add-1 (cons 1 x)) = ??                        {add11}
```


Bignum adder numerals of unbounded length

Simple problem to start: increment by 1

```
; (add-1 x) = numeral for (+ 1 (numb x)))
; (add-1 nil) = (list 1)                        {add1nil}
; (add-1 (cons 0 x)) = (cons 1 x)                {add10}
; (add-1 (cons 1 x)) = (cons 0 (add-1 x))        {add11}
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; (add-1 nil) = (list 1)                        {add1nil}
; (add-1 (cons 0 x)) = (cons 1 x)                {add10}
; (add-1 (cons 1 x)) = (cons 0 (add-1 x))        {add11}
(defun add-1 (x)
  (if (and (consp x) (= (first x) 1))
      (cons 0 (add-1 (rest x))) ; add11
      (cons 1 (rest x))))       ; add10
```


Bignum adder numerals of unbounded length

Simple problem to start: increment by 1

```
; (add-1 x) = numeral for (+ 1 (numb x)))
; (add-1 nil) = (list 1)                        {add1nil}
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(defun add-1 (x)
  (if (and (consp x) (= (first x) 1))
      (cons 0 (add-1 (rest x))) ; add11
      (cons 1 (rest x))))       ; add10
```

Now, add a carry bit c to a numeral

```
; (add-c c x) = numeral for (+ c (numb x)))
(defun add-c (c x)
  (if (= c 1)
      (add-1 x)  ; addc1
      x))        ; addc0
```


Bignum adder numerals of unbounded length

Add with carry – definitional properties

```
; adder with unbounded precision
; (add c0 x y) = numeral for (+ c0 (numb x) (numb y))
; Note: (len x) may be different from (len y)
```

Bignum adder numerals of unbounded length

Add with carry – definitional properties

```
; adder with unbounded precision
; (add c0 x y) = numeral for (+ c0 (numb x) (numb y))
; Note: (len x) may be different from (len y)
(defun add (c0 x y)
  (if (not(consp x))
      (add-c c0 y)                            ; add0y
      (if (not(consp y))
          (add-c c0 x)                        ; addx0y
```

properties that hold when one of the numerals is empty

… other properties (x and y non-nil) …

Bignum adder numerals of unbounded length

Add with carry – definitional properties

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; Note: (len x) may be different from (len y)
(defun add (c0 x y)
  (if (not(consp x))
      (add-c c0 y)                            ; add0y
      (if (not(consp y))
          (add-c c0 x)                        ; addx0y
          (let* ((x0 (first x)) ; x is not nil
                 (y0 (first y)) ; y is not nil
                 (a  (full-adder c0 x0 y0))
                 (s0 (first a))
                 (c1 (second a)))
```

extract low-order bits and add them

Bignum adder numerals of unbounded length

Add with carry – definitional properties

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; (add c0 x y) = numeral for (+ c0 (numb x) (numb y))
; Note: (len x) may be different from (len y)
(defun add (c0 x y)
  (if (not(consp x))
      (add-c c0 y)                            ; add0y
      (if (not(consp y))
          (add-c c0 x)                        ; addx0y
          (let* ((x0 (first x)) ; x is not nil
                 (y0 (first y)) ; y is not nil
                 (a  (full-adder c0 x0 y0))
                 (s0 (first a))
                 (c1 (second a)))
            (cons s0 (add c1 (rest x) (rest y))))))) ; addxy
```

insert low-order sum-bit into numeral
for carry added to high-order bits

Bignum adder numerals of unbounded length

Add with carry – definitional properties

```
; adder with unbounded precision
; (add c0 x y) = numeral for (+ c0 (numb x) (numb y))
; Note: (len x) may be different from (len y)
(defun add (c0 x y)
  (if (not(consp x))
      (add-c c0 y)                            ; add0y
      (if (not(consp y))
          (add-c c0 x)                        ; addx0y
          (let* ((x0 (first x)) ; x is not nil
                 (y0 (first y)) ; y is not nil
                 (a  (full-adder c0 x0 y0))
                 (s0 (first a))
                 (c1 (second a)))
            (cons s0 (add c1 (rest x) (rest y))))))) ; addxy
```

correctness property

(numb(add c0 x y)) = c0 + (numb x) + (numb y)

demo 7

Mechanization Is Necessary

without it, all is lost in the details

Even simple properties lead to big proofs

- ✓ millions of details in proofs of software properties
  - People can't keep track of millions of details
  - Besides, a proof at least is as likely to be wrong as a program
- ✓ people formulate properties … computers push details
  - proof organized into lemmas — similar to software components
    - rigorous, but not fully formal
    - like a paper-and-pencil proof, as done by mathematicians
  - some lemma architectures are better than others
    - like modular decomposition of software … design matters
  - formulation of properties is a big task
    - experience/judgment required … as in software development

When is theorem-proving practical?

Mission-critical: defect would be a catastrophe

- ✓ Intel Pentium bug in floating-point division
  - convinced AMD to spend 12 weeks with ACL2 team
  - AMD test suite for that circuit had 80-million cases
  - gazillions of potential cases ($2^{15+64} \times 2^{15+64} = 2^{158}$)
  - physically impossible to do that many tests
- ✓ NSA apparently willing to make large investments to eliminate the possibility of certain outcomes

Specifiable properties

- ✓ Catastrophe avoided when Boolean formula holds

Resources

- ✓ VLSI design: add 10% to project budget/schedule
- ✓ software: double, triple, … or more

Exercises - bignum multiplier

4. Verify: numb inverts bits
5. Write defining properties for a multiplication operator for binary numerals of unbounded length
6. Define a correctness property of your multiply op
7. Use Proof Pad to run tests of the property you defined
8. Verify that the property holds for all binary numerals

hints

shift and add

✓ x_0 + 2∗(numb x) = (numb(cons x_0 x))

natural numbers

(natp x) ≡ x ∈ {0, 1, 2, …}

x is even

✓ $x = 2\lfloor x/2 \rfloor$

✓ $xy = 2\lfloor x/2 \rfloor y$

x is odd

✓ $x = 1 + 2\lfloor x/2 \rfloor$

✓ xy = (mod y 2) + 2($\lfloor y/2 \rfloor$ + $\lfloor x/2 \rfloor$y)

Notes: http://ceres.hh.se/mediawiki/index.php/HSST_2014

Install Proof Pad: http://proofpad.org

The End

June 12, 2014

11:00-12:30 session