

Path Testing

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<http://bit.ly/TAV16>

Testing and Verification,
February 12, 2016

Outline

Structural Testing: An Introduction

Flow Graphs

Path Testing

Prime Paths

Testability

Functional Testing: Pros and Cons

Pros:

- ▶ Straightforward test-case generation
- ▶ Based on specification (early test-case generation)

Cons:

- ▶ No use of program information
- ▶ Gaps and redundancies

Structural Testing

Idea

- ▶ Derive **structural abstractions** from programs
Example: **flow graphs**
- ▶ Use them to **measure** the **adequacy** of the test-set

Structural Testing (Example from the 1st Lecture)

Spec.: input: an integer x [$1..2^{16}$]

output: x incremented by two, if x is less than 50,
 x decremented by one, if x is greater than 50, and
50, otherwise.

```
if  $x < 50$  then
```

```
   $x = x + 2$ ;
```

```
end if
```

```
if  $x > 50$  then
```

```
   $x = x - 1$ ;
```

```
end if
```

```
return  $x$ 
```

Structural Testing

```
if  $x < 50$  then  
   $x = x + 2$ ;  
end if  
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   $x = x - 1$ ;  
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return  $x$ 
```

Adequacy criterion: test until all statements are at least executed once (subject of today's lecture: **DD-path coverage**).

Structural Testing

```
if  $x < 50$  then  
   $x = x + 2$ ;  
end if  
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end if  
return  $x$ 
```

Adequacy criterion: test until all statements are at least executed once (subject of today's lecture: **DD-path coverage**).

Input	Output	Pass/Fail
3222	3221	P
30	32	P

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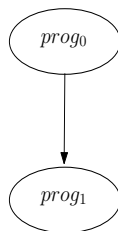
Flow Graphs

- ▶ Nodes: program statements
- ▶ Edges: $p \rightarrow q$ iff q may execute immediately after p

From Programs to Flow Graphs: Examples

Flow Graph for simple statements

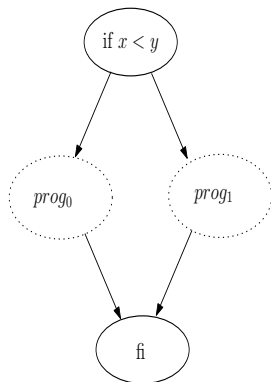
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prog₀; prog₁,



From Programs to Flow Graphs: Examples

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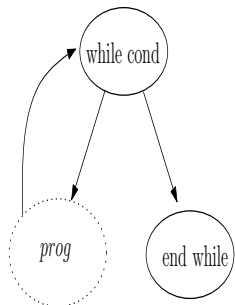
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 $if(cond) then prog_0 else prog_1 fi,$



From Programs to Flow Graphs: Examples

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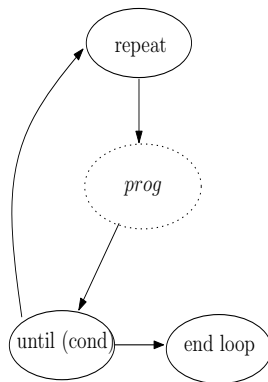
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prog₀; prog₁,
- ▶ Conditional:
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- ▶ While loop:
while(cond) do prog endwhile,



From Programs to Flow Graphs: Examples

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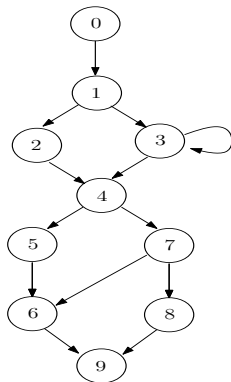
- ▶ Sequential composition:
prog₀; prog₁,
- ▶ Conditional:
if(cond) then prog₀ else prog₁ fi,
- ▶ While loop:
while(cond) do prog endwhile,
- ▶ Repeat-until loop:
repeat prog until(cond),



Test Adequacy Criteria

The **test-set** covers, in the flow graph,

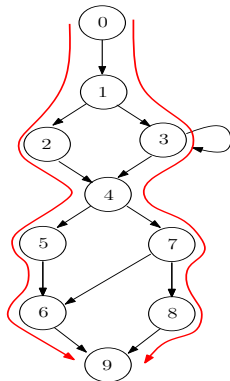
1. **all nodes** (statement coverage)



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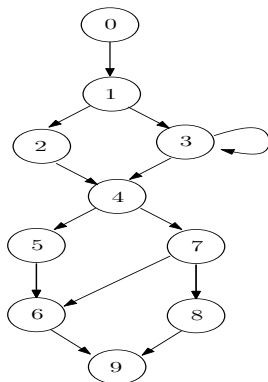
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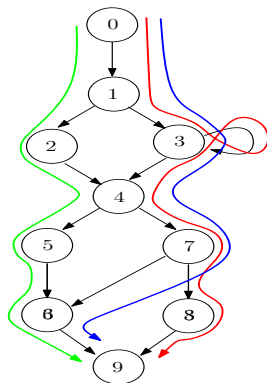
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2. **all edges** (DD-path coverage)



Test Adequacy Criteria

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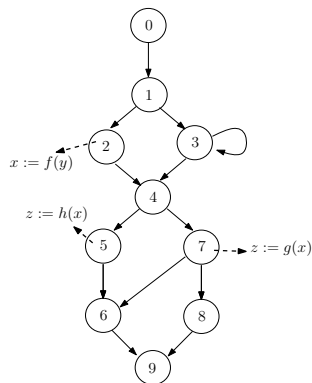
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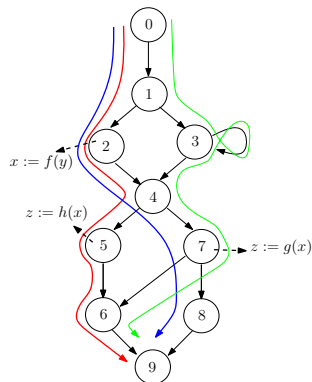
1. **all nodes** (statement coverage)
2. **all edges** (DD-path coverage)
3. **all prime paths** (single-loop coverage)
4. **all edges + all combinations of data-flow dependent edges** (dependent pairs coverage: **next lecture**)



Test Adequacy Criteria

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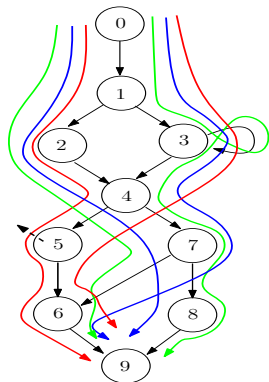
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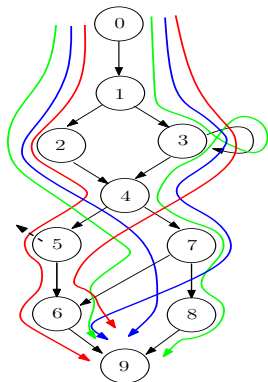
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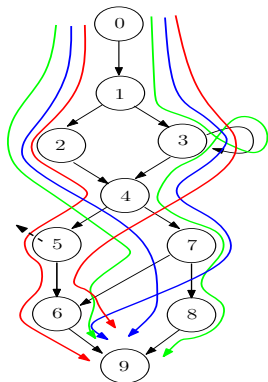
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3. **all prime paths** (single-loop coverage)
4. **all edges** + **all combinations** of data-flow **dependent edges** (dependent pairs coverage)
5. **all edges** + **all combinations** of **condition edges** (multiple-condition coverage)
6. **all paths** (full path coverage)



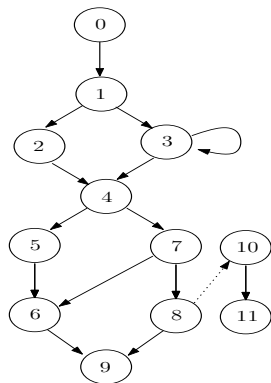
Finite Feasibility

An adequacy criteria should be **satisfiable** by some **finite test-set**.

Question: Which of the aforementioned criteria are finitely feasible?

Finite Feasibility

An adequacy criteria should be satisfiable by some finite test-set.



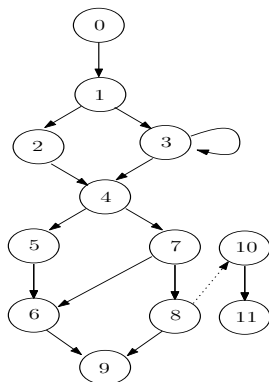
Finite Feasibility

An adequacy criteria should be satisfiable by some finite test-set.

Solution: Adding **feasibility**:

1. **all reachable nodes** (feasible statement coverage)
2. **all reachable edges** (feasible DD-path coverage)
3. **all reachable ...**

Problem solved?



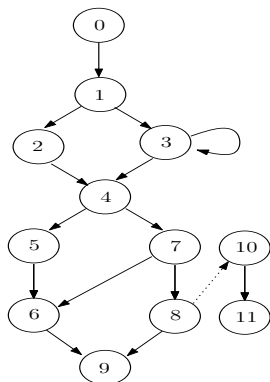
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1. **all reachable nodes** (feasible statement coverage)
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Problem solved? No, checking **reachability** is **undecidable** in general!



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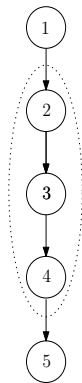
Testability

Chain: Definition

A chain n_0, \dots, n_i , with $0 \leq i$, is a list of nodes s.t.

1. $n_j \rightarrow n_{j+1}$ for each $j < i$,
2. $\text{indeg}(n_j) = \text{outdeg}(n_j) = 1$, for each $0 \leq j \leq i$,

A chain n_0, \dots, n_i is **maximal** when neither n', n_0, \dots, n_i nor n_0, \dots, n_i, n' (for any n') are chains. Each **node** is a member of **at most one maximal chain**.



DD-Path: Definition

A DD-Path is a set of nodes satisfying one of the following:

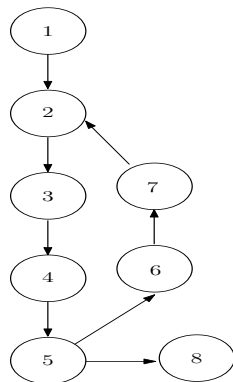
1. $\{n\}$ s.t. $indeg(n) = 0$ (**starting** node) or $outdeg(n) = 0$ (**terminal** node),
2. $\{n\}$ s.t. $outdeg(n) \geq 2$ or $indeg(n) \geq 2$ (**branch** or **merge** nodes)
3. $\{n_0, \dots, n_i\}$ with $i \geq 0$ s.t.
 $n_0 \rightarrow \dots \rightarrow n_i$ is a **maximal chain**

Property: each **node** belongs to precisely **one DD-path**

DD-Path: Simplified Definition

A DD-Path is a set of nodes satisfying one of the following:

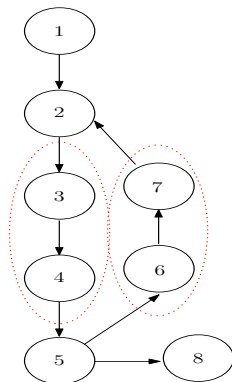
1. $\{n\}$ s.t. $indeg(n) \neq 1$ or $outdeg(n) \neq 1$,
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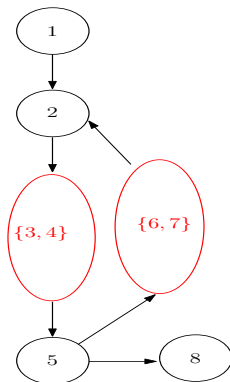
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DD-Path Graph

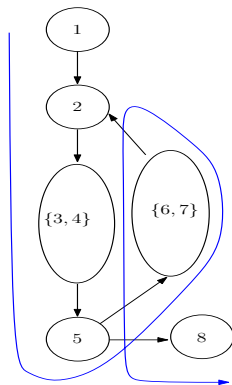
In a DD-Path graph:

1. nodes: DD-Paths as
2. edges: $\{n_i \mid i \in I\} \rightarrow \{m_j \mid j \in J\}$
when $\exists i' \in I, j' \in J$ s.t.
 $n_{i'} \rightarrow m_{j'}$.



DD-Path Coverage

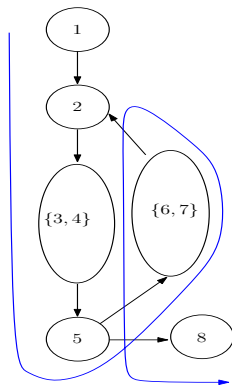
A test-set is adequate when for each **node or edge** in the DD-path graph, there exists a test-case covering it.



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This is equivalent to edge coverage, but requires less checks.

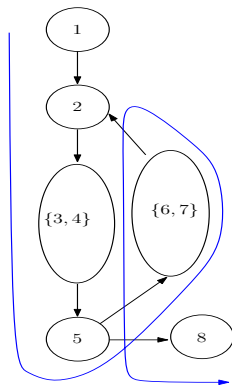


DD-Path Coverage

A test-set is adequate when for each **node or edge** in the DD-path graph, there exists a test-case covering it.

This is equivalent to edge coverage, but requires less checks.

This subsumes node coverage.



DD-Path Testing: Complete?

```
if  $x < 50$  then  
   $x = x + 2$ ;  
end if  
if  $x > 50$  then  
   $x = x - 1$ ;  
end if  
return  $x$ 
```

Input	Output	Pass/Fail
3222	3221	P
30	32	P
49	51	F
50	50	P

DD-Path: Complete?

Solutions:

1. Use **stronger adequacy** criteria: prime paths, dependent pairs testing, multiple condition coverage testing
2. Problems: more test-sets; even sometimes: not that many more faults detected
3. Use more **switch** statements instead of sequential conditions.

DD-Path Testing

Pros:

1. DD-paths instead of statements: more **efficient coverage measuring**
2. DD-paths coverage: a practical measure of test **adequacy**
3. **implemented** in many tools

Cons:

1. **infeasible** paths must be tested!
2. some important **paths** left **untested**
3. no **test-case generation** technique
4. main reason: **ignoring specification** and **data**-dependencies: dependent pairs testing (see the next lecture)

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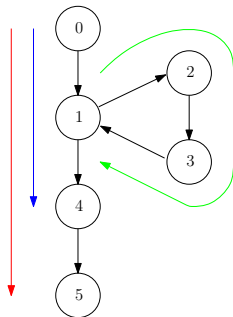
Testability

Simple Path: Definition

A **simple** path n_0, \dots, n_t , with $0 \leq t$, is a list of nodes s.t.

1. $n_j \rightarrow n_{j+1}$ for each $j < t$,
2. for each $0 \leq i < j \leq t$, $n_i \neq n_j$
or ($n_i = n_0$ and $n_j = n_t$)

Informally: a simple path visits a node **at most once**, except that the start and the ending node may be **the same**.

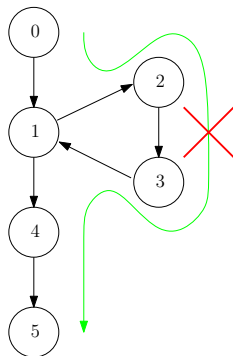


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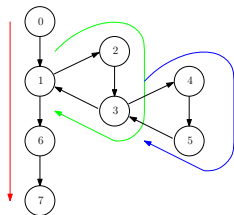


Prime Path: Definition

A **prime** path is:

- ▶ a **simple** path that
- ▶ does not appear as a **proper sub-path** of any other **simple** path.

Informally: a prime path is a complete path from start to end, or a complete and simple iteration of a loop (infeasibility issue set aside)

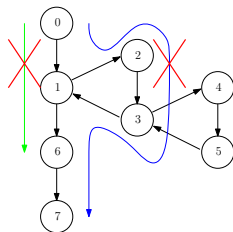


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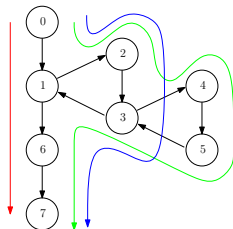
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Prime Path Coverage

A test set is adequate if for each prime path, there is a test case covering it (as a sub-path).

Informally: all complete simple paths and **up to one** iteration of each loop

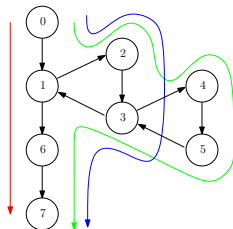


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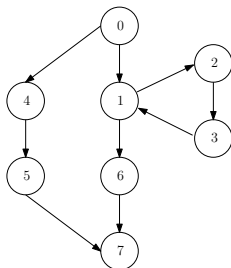
Informally: all complete simple paths and **up to one** iteration of each loop

Variants with tours, detours and side-trips



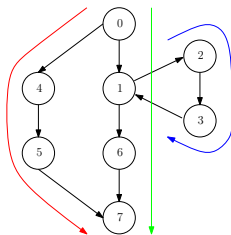
Prime Path Coverage: Exercise

Propose a set of test cases that is adequate for prime path coverage.



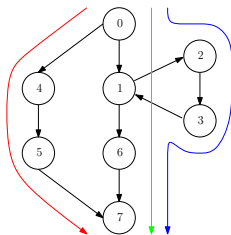
Prime Path Coverage: Solution

Prime paths



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Prime paths



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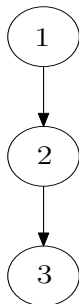
Testability

Testability: Cyclomatic number

1. Idea (very informal):
Take one path from start to exit,
count the number of alternatives by
flipping one condition at a time.
2. Also called: nullity, first Betti number, dimension of cycle space

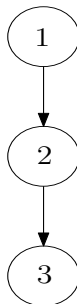
Cyclomatic number: Examples

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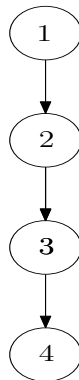
Cyclomatic number: Examples

Cyclomatic number: **1**



Cyclomatic number: Examples

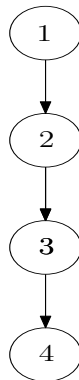
Cyclomatic number:



Cyclomatic number: Examples

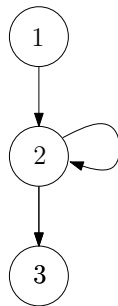
Cyclomatic number: **1**

Observation: Cyclomatic nr. is **size independent...**



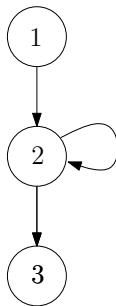
Cyclomatic number: Examples

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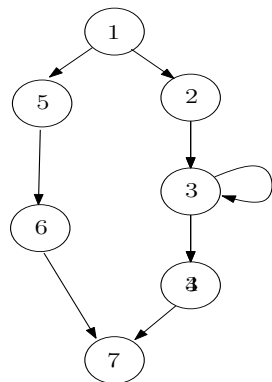
Cyclomatic number: Examples

Cyclomatic number: 2



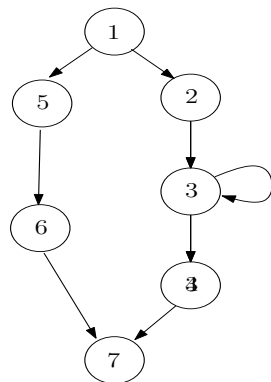
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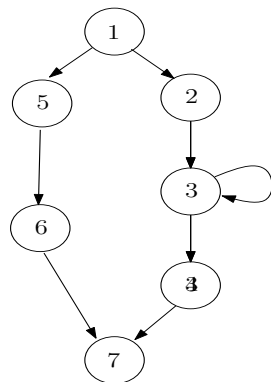
Cyclomatic number: Examples

Cyclomatic number: 3



Cyclomatic number: Examples

Cyclomatic number:



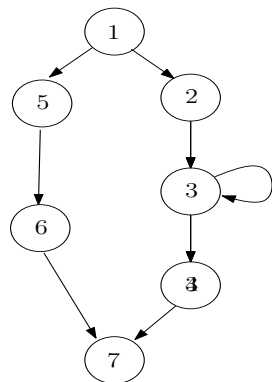
Cyclomatic number: Examples

Cyclomatic number: 3

Only for programs with:

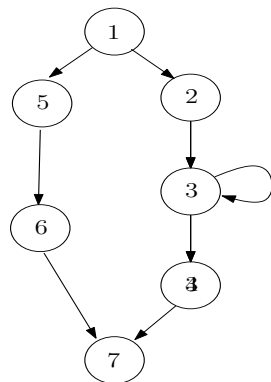
1. one **connected component**,
2. one **starting state**, and
3. one **terminal state**:

$$V(G) = \#edges - \#nodes + 2$$



Cyclomatic number: Calculation

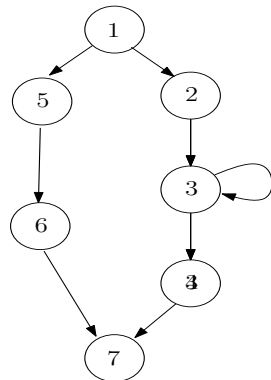
Cyclomatic number:



Cyclomatic number: Calculation

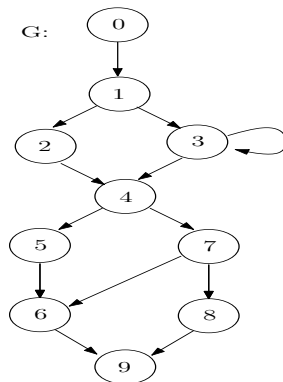
Cyclomatic number: 3

For (connected) planar graphs:
 $V(G) = \#regions\ in\ the\ plane$



Cyclomatic number: Example

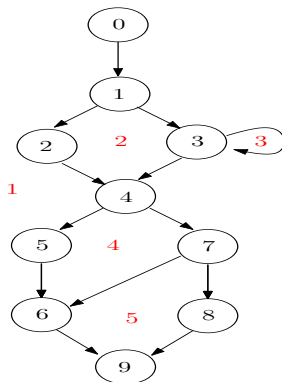
1. the cyclomatic number of G ?
2. the cyclomatic number of G **sequentially composed** with itself?
3. the cyclomatic number of G **n -times** sequentially composed with itself?



Cyclomatic number: Example

What is

1. the cyclomatic number of G ? **5**
2. the cyclomatic number of G sequentially composed with itself? **$9 = 5 + 5 - 1$**
3. the cyclomatic number of G n -times sequentially composed with itself? **$4 * n + 1$**



Cyclomatic number: Implications

Cyclomatic Complexity	Risk Evaluation
1-10	a simple program, without much risk
11-20	more complex, moderate risk
21-50	complex, high risk program
>50	untestable program (very high risk)

Conclusions

1. Cyclomatic number: a measure for software **complexity** and **testability**
watch out for programs with $V(G) > 10!$
2. **implemented** in several tools (see the course web-site for examples)
3. a measure of **test-set adequacy** (originally invented for this purpose!)

Conclusions

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watch out for programs with $V(G) > 10!$
2. **implemented** in several tools (see the course web-site for examples)
3. a measure of **test-set adequacy** (originally invented for this purpose!)
Let's have a better look...