System Validation: Parallel Processes

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Motivation
Decomposing a Two-Place Buffer
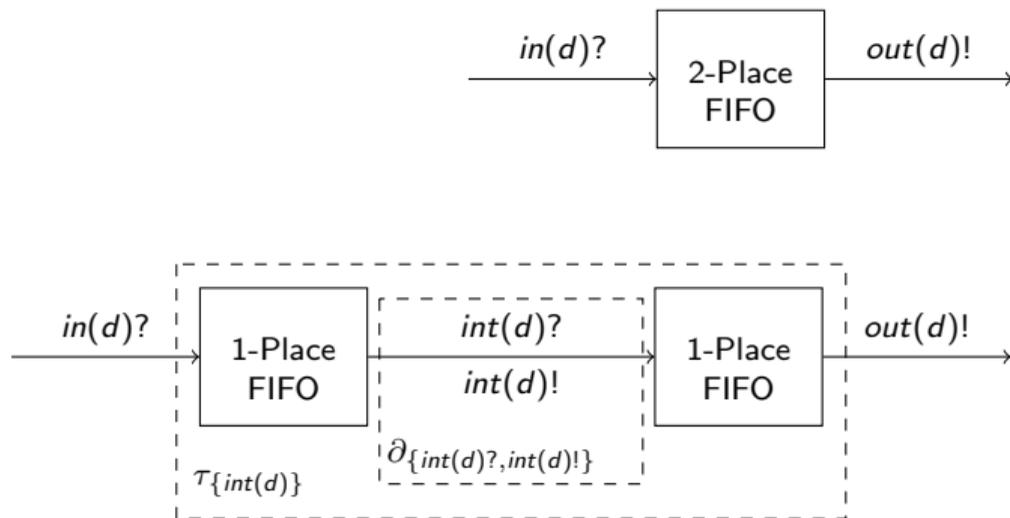
Parallel Composition

Semantics of $p \,\|\, q$

Parallel Composition

Semantics of $p \,||\, q$

- $p \xrightarrow{\alpha} p'$, then $p \,||\, q \xrightarrow{\alpha} p' \,||\, q$,
- $q \xrightarrow{\alpha} q'$, then $p \,||\, q \xrightarrow{\alpha} p \,||\, q'$, and

Parallel Composition

Semantics of $p \parallel q$

- $p \xrightarrow{\alpha} p'$, then $p \parallel q \xrightarrow{\alpha} p' \parallel q$,
- $q \xrightarrow{\alpha} q'$, then $p \parallel q \xrightarrow{\alpha} p \parallel q'$, and
- $p \xrightarrow{\alpha} p'$ and $q \xrightarrow{\beta} q'$, then $p \parallel q \xrightarrow{\alpha|\beta} p' \parallel q'$.
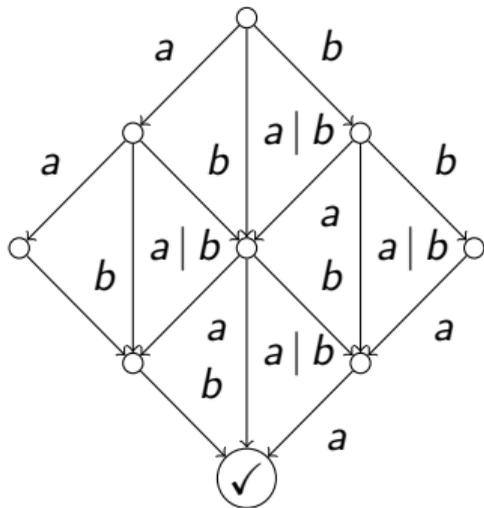
$a \cdot a \,||\, b \cdot b$

The Saga of Axiomatizing Parallel Composition

Challenge

$$(Dish1 + Dish2) \parallel Coke \stackrel{?}{=} (Dish1 \parallel Coke) + (Dish2 \parallel Coke)$$

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Faron Moller's Result

Parallel composition (without auxiliary operators)
cannot be finitely axiomatized.

The Saga of Axiomatizing Parallel Composition

Bergstra and Klop's Invention

Axiomatize parallel composition using:

- Left merge ($\parallel$):
  $p \xrightarrow{\alpha} p'$, then $p \parallel q \xrightarrow{\alpha} p' \parallel q$,

- Communication merge ($|$):
  $p \xrightarrow{\alpha} p'$ and $q \xrightarrow{\beta} q'$, then $p \mid q \xrightarrow{\alpha|\beta} p' \parallel q'$.

The Saga of Axiomatizing Parallel Composition

Bergstra and Klop's Invention

Axiomatize parallel composition using:

- Left merge ($\parallel$):
  $p \xrightarrow{\alpha} p'$, then $p \parallel q \xrightarrow{\alpha} p' \parallel q$,
- Communication merge ($\mid$):
  $p \xrightarrow{\alpha} p'$ and $q \xrightarrow{\beta} q'$, then $p \mid q \xrightarrow{\alpha\mid\beta} p' \parallel q'$.

Expansion Law

$$p \parallel q = (p \parallel q) + (q \parallel p) + (p \mid q)$$

The Saga of Axiomatizing Parallel Composition

$\parallel$ and $\mid$: Raisons d'être

$(Dish1 + Dish2) \parallel Coke \underline{\leftrightarrow} (Dish1 \parallel Coke) + (Dish2 \parallel Coke)$

$(Dish1 + Dish2) \mid Coke \underline{\leftrightarrow} (Dish1 \mid Coke) + (Dish2 \mid Coke)$

Parallel Composition
Axioms for Untimed Processes

M $\quad x \parallel y = x \rLfloor y + y \rLfloor x + x \mid y$

LM1 $\quad \alpha \rLfloor x = \alpha{\cdot}x$
LM2 $\quad \delta \rLfloor x = \delta$
LM3 $\quad \alpha{\cdot}x \rLfloor y = \alpha{\cdot}(x \parallel y)$
LM4 $\quad (x + y) \rLfloor z = x \rLfloor z + y \rLfloor z$
LM5 $\quad (\sum_{d:D} X(d)) \rLfloor y = \sum_{d:D} X(d) \rLfloor y$

Parallel Composition
Axioms for Untimed Processes (cont'd)

S1 $x \mid y = y \mid x$
S2 $(x \mid y) \mid z = x \mid (y \mid z)$
S3 $x \mid \tau = x$
S4 $\alpha \mid \delta = \delta$
S5 $(\alpha{\cdot}x) \mid \beta = \alpha \mid \beta{\cdot}x$
S6 $(\alpha{\cdot}x) \mid (\beta{\cdot}y) = \alpha \mid \beta{\cdot}(x \parallel y)$
S7 $(x + y) \mid z = x \mid z + y \mid z$
S8 $(\sum_{d:D} X(d)) \mid y = \sum_{d:D} X(d) \mid y$

TC1 $(x \mathbin{\|\!\|} y) \mathbin{\|\!\|} z = x \mathbin{\|\!\|} (y \parallel z)$
TC2 $x \mathbin{\|\!\|} \delta = x{\cdot}\delta$
TC3 $(x \mid y) \mathbin{\|\!\|} z = x \mid (y \mathbin{\|\!\|} z)$

Thank you very much.