DT8014 Algorithms (2016)
Week 2 - Algorithm Complexity and Graphs
Activity 2
September 7, 2016

Working with Graph Properties

Work together on the following tasks within your group.

Task 1

1. Draw all planar bipartite graphs where one of the two vertex sets contains 3 vertices and each vertex from one set is adjacent to all the vertices of the other.
   After 5 minutes, present your examples on the whiteboard.

2. Sketch an algorithm that determines whether a given graph is 2-connected.
   After 10 minutes, present your algorithms on the whiteboard.

Task 2

1. Draw a non-planar graph whose maximal clique size is 2.
   After 5 minutes, present your example(s) on the whiteboard.

2. Sketch an algorithm that determines whether a given graph is 2-connected.
   After 10 minutes, present your algorithms on the whiteboard.

Task 3

1. Draw a simple 4-regular graph whose maximal clique size is 4.
   After 5 minutes, present your example(s) on the whiteboard.

2. Sketch an algorithm that determines whether a given graph is bipartite.
   After 10 minutes, present your algorithms on the whiteboard.

Task 4

1. Draw a 2-connected simple graph, and a planar graph with a 4-clique.
   After 5 minutes, present your examples on the whiteboard.

2. Sketch an algorithm that determines whether a given graph is bipartite.
   After 10 minutes, present your algorithms on the whiteboard.

Hamiltonian Graphs

Form groups of 2-3 students and work together on the following tasks.

A **Hamiltonian path** is a path in an undirected or directed graph G that visits each node exactly once. A **Hamiltonian cycle** is a Hamiltonian path that is a cycle, i.e. it starts and ends at the same node. A graph that contains a Hamiltonian cycle is called a **Hamiltonian graph**. Figure 1 shows an example graph with a Hamiltonian cycle[1].

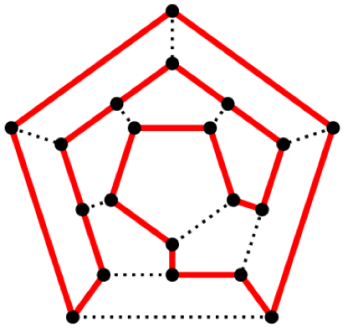

Figure 1: Hamiltonian cycle

1. Create two examples of Hamiltonian graphs. Then, create two examples of non-Hamiltonian (but connected) graphs.

2. For the graph shown in Figure 2, list all the paths that start at node A and visit each node at most once (i.e. some nodes may remain unvisited). Which of these paths are Hamiltonian? Which of these paths can be completed into a Hamiltonian cycle?

3. Suppose you are given a graph G and a sequence of nodes L. How would you check whether L is a Hamiltonian path in G? How many steps would your method require –in the worst case– for a graph with N nodes?

[1] Taken from http://en.wikipedia.org/wiki/Hamiltonian_path

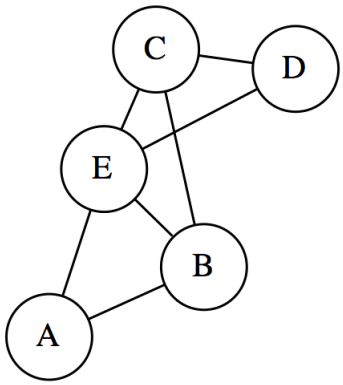

Figure 2: Find all Hamiltonian paths starting at A and all Hamiltonian cycles in this graph.