From partial evaluation to algebra using first principles

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IFIP WG 2.11
Program Generation

Delft
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supported by: THE ROYAL SOCIETY The Alan Turing Institute Facebook Research typical concepts in

Partial Evaluation:

- interpreters/compilers
- rewriting, optimization
- data structures
- algorithmics

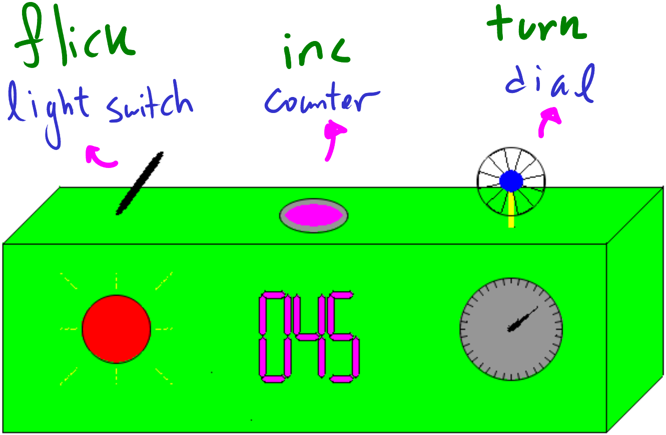
Algebra:

- polynomials
- axioms
- solving equations

Goal:

- Implement Partial Evaluator
- Recover algebra

Vehicle: Action Box



Key Idea

Partial Evaluation

$\Updownarrow$

Evaluation in abstract domain

Established Idea

Partial Evaluation

$\Updownarrow$

Evaluation in abstract domain

- Abstract interpretation

  Here: accuracy

- Denotational semantics

  Here: implementability/effectivity

Algebraic Perspective

$$A := (\mathrm{Cmds}, \mathrm{State}, \mathrm{act})$$

monoid actions

script vars

state vars

$$A[X, Y] := (\mathcal{O}[X,Y], S[X,Y], \mathrm{act})$$

We have:

$$A \xrightarrow{\mathrm{sta}} A[X,Y] \xleftarrow{\mathrm{dyn}} \langle X, Y \rangle$$

also:

homomorphism

(respects action & sequencing)

Def. Extension of action A by sets $\langle X, Y\rangle$:

another action — action homomorphism

$$A \xrightarrow{h} B \xleftarrow{e} \langle X, Y\rangle$$

Fact: Representation theorem — inductive/non-quotiented implementation

$$A[X,Y] := (\mathrm{ROScript}[X], \mathrm{RoState}[X,Y])$$

is the free extension:

$$A \xrightarrow{\mathrm{sta}} A[X,Y] \xleftarrow{\mathrm{dyn}} \langle X, Y\rangle$$

Key idea: Generalise action

multi-sorted algebra

Def. Extension of algebra A by sets $\langle X, Y \rangle$:

algebra   algebra homomorphism

$$A \xrightarrow{h} B \xleftarrow{e} \langle X, Y \rangle$$

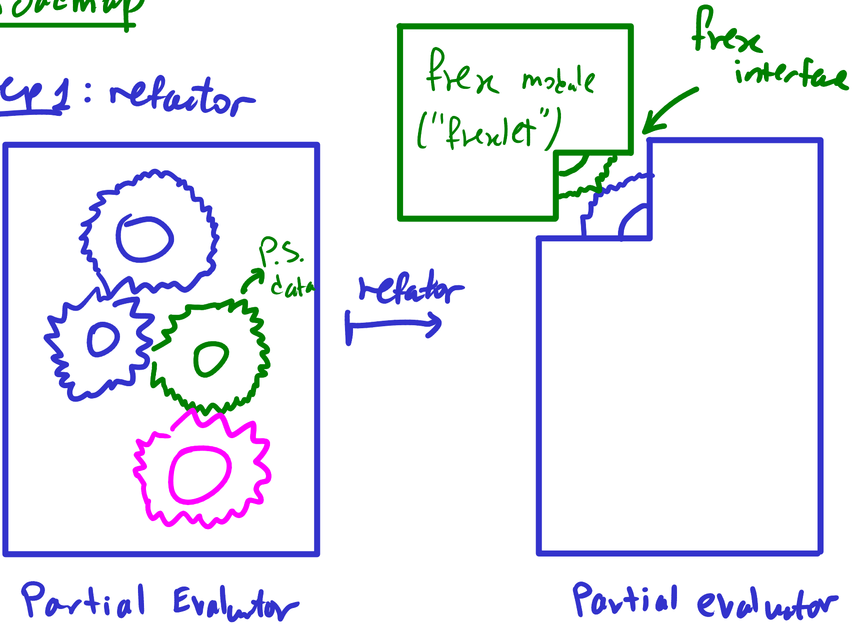
inductive/non-quotiented implementation

Goal: Representation theorems for the free extension:

$$A \xrightarrow{sta} A[X, Y] \xleftarrow{dyn} \langle X, Y \rangle$$

Roadmap

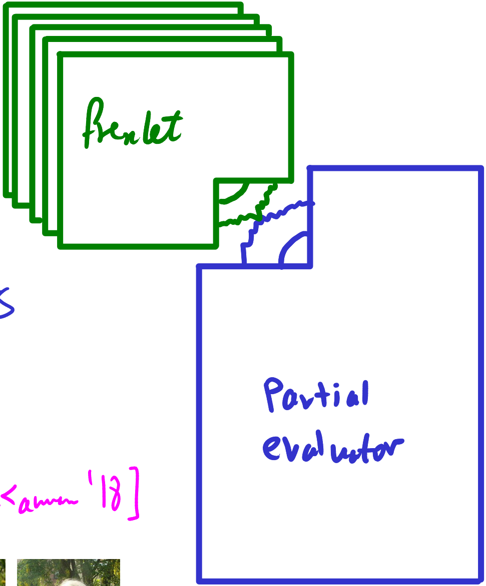
Step 1: refactor



Roadmap

Stepz:

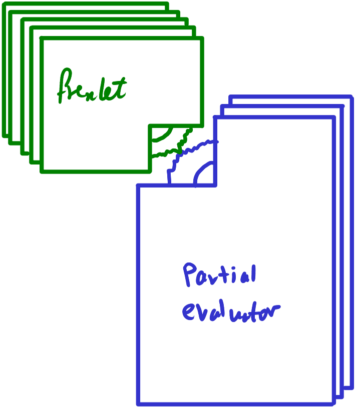
develop multiple frexlets



Staging with Frex

[Yallop, von Glehn, Kammar '18]

Roadmap



Step 3:

develop generic partial evaluators:

- ▷ Staging-based optimisation [Yallop, van Glehn, Kammar '18]

- ▷ equational proof synthesis [Allais, Brady, Corbyn, Kammar, Yallop, ongoing]

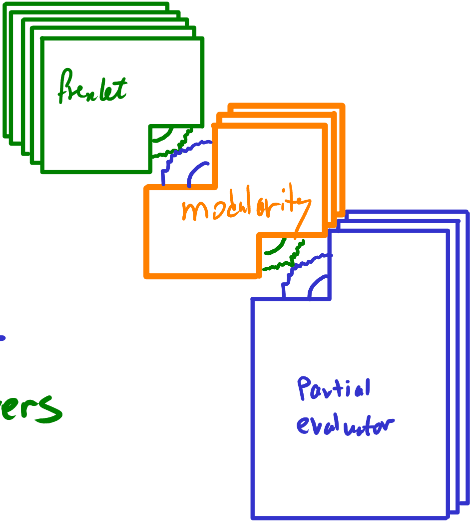
- ▷ Normalization-by-Evaluation [Corbyn, Kammar, Lindley, Valliappan, Yallop ongoing]

Roadmap

Step3:

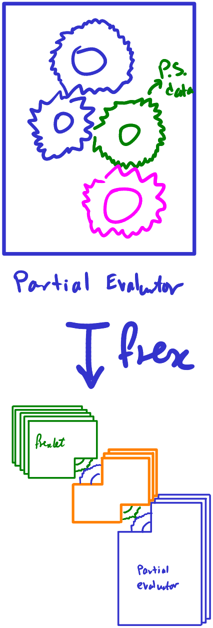
modular construction of

- involutive algebra frex
- homomorphism graphs frex
- (ongoing) ring solvers



Conclusion

- ▷ Partial evaluation $\rightsquigarrow$ Algebra
- ▷ a roadmap:
  - staging
  - equational-proof synthesis
  - NbE
  - Modularity



Thanks!