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Farms, Pipes, Streams and Reforestation

Type-Directed Parallelisation

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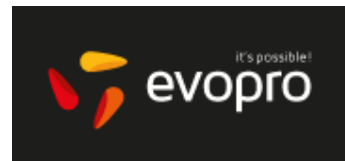
IFIP Working Group 2.11 Meeting, Bloomington, Indiana, 23/8/16



RePhrase Project: Refactoring Parallel Heterogeneous Software – a Software Engineering Approach (ICT-644235), 2015-2018, €3.6M budget

8 Partners, 6 European countries
UK, Spain, Italy, Austria, Hungary, Israel

Coordinated by Kevin Hammond, St Andrews



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ParaFormance Project: Parallel Patterns for Heterogeneous Multicore Systems (ICT-288570), 2015-2018, £537K budget

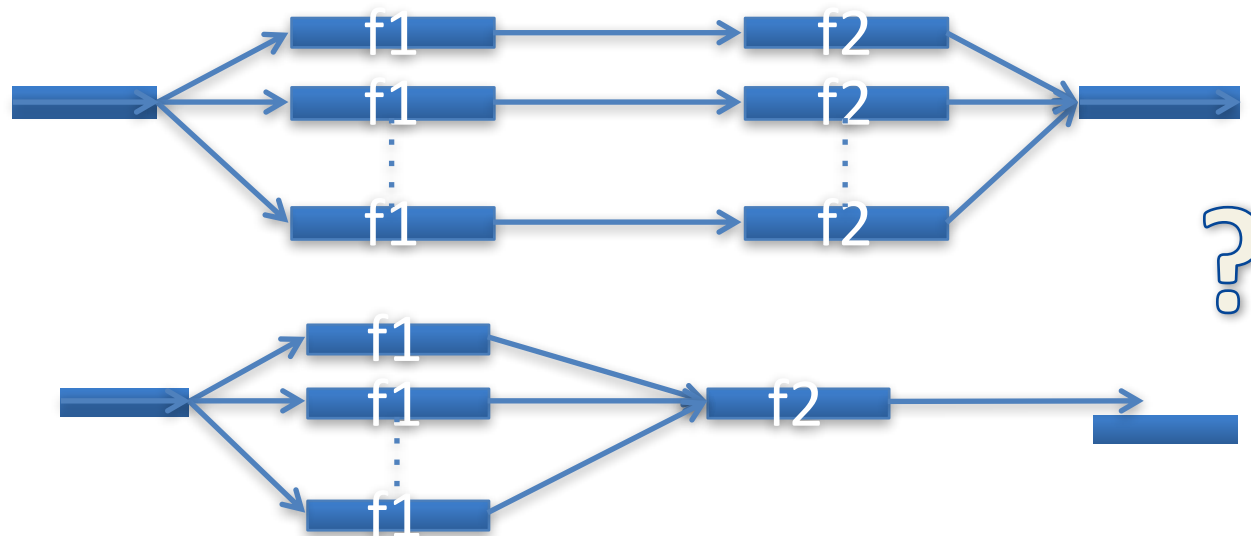
Goal: Formation of High-Growth Company of Scale by 2023

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The Problem

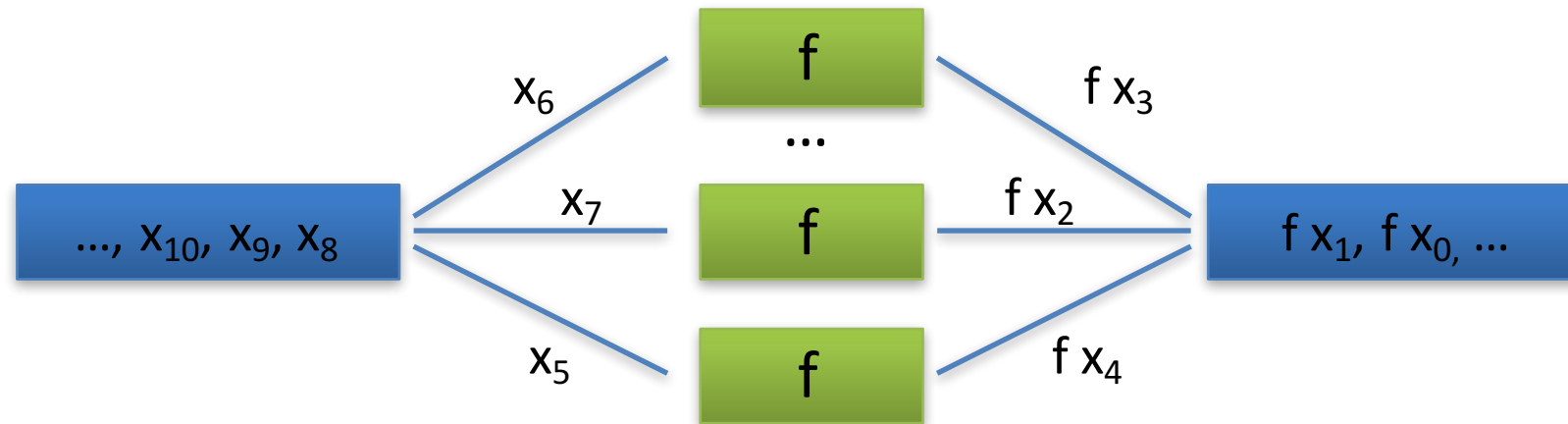
- We need to choose the best parallel **abstractions**
 - *Algorithmic skeletons* [Cole 1989] implement patterns
- We need a formal way to reason about parallel **structure**
 - Correctness of transformations
 - Reasoning about performance





Example Skeleton: Parallel Task Farms

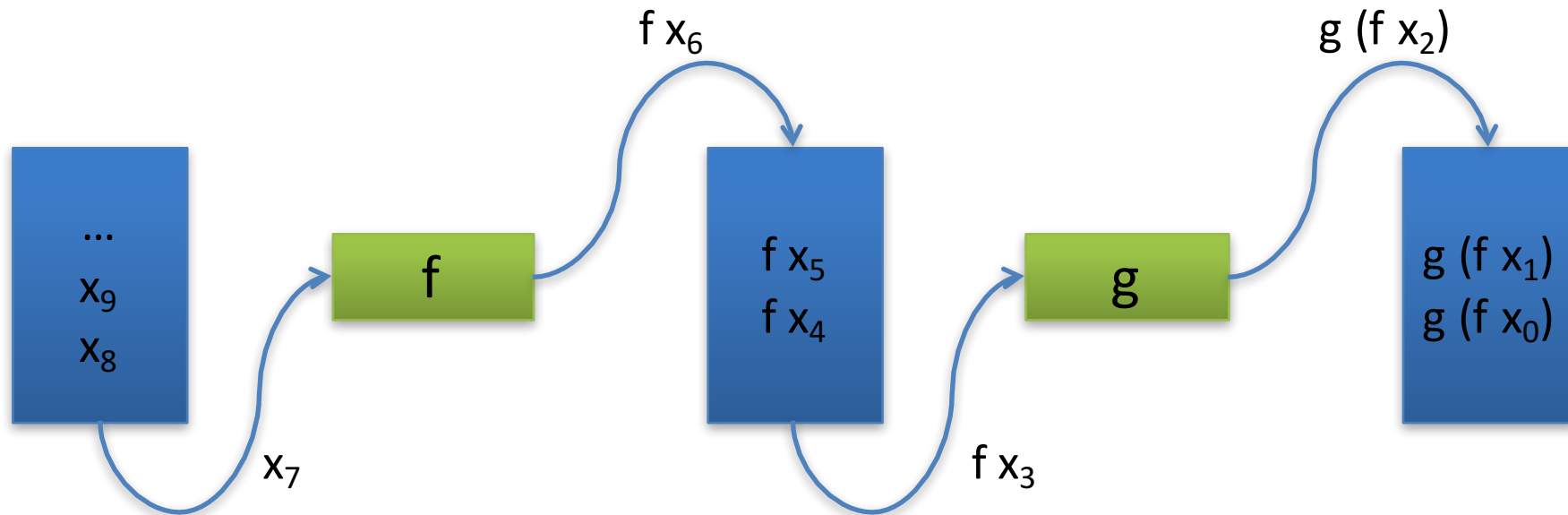
- Task Farms use a fixed number of workers
 - Each worker applies the same operation (f)
 - f is applied to each of the inputs in a stream.





Example Skeleton: Parallel Pipeline

- Parallel pipelines compose two operations (**f** and **g**)
 - over the elements of an input stream
 - f** and **g** are run in parallel





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Example

Image Merge

Image merging composes two operations, **merge** and **mark**

$$\text{imgMerge} : \text{List}(\text{Img} \times \text{Img}) \rightarrow \text{List } \text{Img}$$
$$\text{imgMerge} = \text{map} (\text{merge} \circ \text{mark})$$

Possible implementations include:

$$\begin{aligned} \text{imgMerge}_1 &= \text{farm } n (\text{fun } (\text{merge} \circ \text{mark})) \\ \text{imgMerge}_2 &= \text{farm } n (\text{fun } \text{mark}) \parallel \text{farm } m (\text{fun } \text{merge}) \\ \text{imgMerge}_3 &= \text{farm } n (\text{fun } \text{merge}) \parallel \text{fun } \text{mark} \\ &\dots \end{aligned}$$

Choosing an Implementation

Decorate the function type with $IM(n,m)$

$$\begin{aligned} \text{imgMerge} & : \text{List}(\text{Img} \times \text{Img}) \xrightarrow{IM(n,m)} \text{List} \text{Img} \\ \text{imgMerge} & = \text{map} (\text{merge} \circ \text{mark}) \end{aligned}$$

where

$$IM(n, m) = \text{FARM } n (\text{FUN } A) \parallel \text{FARM } m (\text{FUN } A)$$

Now the type system automatically selects

$$\text{imgMerge}_2 = \text{farm } n (\text{fun mark}) \parallel \text{farm } m (\text{fun merge})$$

We can *guarantee* that this is *functionally equivalent* to imgMerge



Inferring parallel structures

We can leave holes in the types, e.g.

$$IM(n,m) = _ || FARM m _$$

replaces $_$ with the simplest possible structures

$$IM(n,m) = \min \text{cost} (_ || FARM m _)$$

replaces $_$ with the least cost structures.

We can choose the provably least cost skeleton



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Basic Semantics

Syntax of Skeletons



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$$p \in P ::= \text{fun}_T f \mid p_1 \parallel p_2 \mid \text{dc}_{n,T,F} f g \mid \text{farm } n p \mid \text{fb } p$$

fun_T lifts an atomic function to a collection type T
 dc represents divide and conquer over collection T
 fb introduces feedback

Skeleton Denotational Semantics

Base semantics, S (ρ is a global environment of function defs)

$$\begin{aligned} S[p : T A \rightarrow T B] & : [A \rightarrow B] \\ S[\text{fun } f] & = \hat{\rho}(f) \\ S[p_1 \parallel p_2] & = S[p_2] \circ S[p_1] \\ S[\text{farm } n \ p] & = S[p] \\ S[\text{fb } p] & = \text{iter } S[p] \\ S[\text{dc}_{n,T,F} f \ g] & = \text{cata}_F (\hat{\rho}(f)) \circ \text{ana}_F (\hat{\rho}(g)) \end{aligned}$$

Lifted to a streaming form, P over collection type T

$$\begin{aligned} [p : T A \rightarrow T B] & : [T A \rightarrow T B] \\ [p] & = \text{map}_T S[p] \end{aligned}$$

Morphisms for Divide-and-Conquer

$$\mathcal{S}[\text{dc}_{n,T,F} f g] = \text{cata}_F (\hat{\rho}(f)) \circ \text{ana}_F (\hat{\rho}(g))$$

Catamorphism (fold)

$$\begin{aligned} \text{cata}_F & : (F A \rightarrow A) \rightarrow \mu F \rightarrow A \\ \text{cata}_F f & = f \circ F (\text{cata}_F f) \circ \text{out}_F \end{aligned}$$

Anamorphism (unfold)

$$\begin{aligned} \text{ana}_F & : (A \rightarrow F A) \rightarrow A \rightarrow \mu F \\ \text{ana}_F g & = \text{in}_F \circ F (\text{ana}_F g) \circ g \end{aligned}$$



Morphisms for Streams

$$\llbracket p \rrbracket = \text{map}_T \mathcal{S} \llbracket p \rrbracket$$

Given a *bifunctor*, G , maps over collection T are

$$\begin{aligned} \text{map}_T f &= \text{cata}_{G A}(\text{in}_{G B} \circ G f \text{id}) \\ &= \text{ana}_{G B}(G f \text{id} \circ \text{out}_{G A}) \end{aligned}$$

Iteration

$$\mathcal{S}[\text{fb } p] = \text{iter } \mathcal{S}[p]$$

Easy to define using the fix-point combinator, $Y f = f (Y f)$

$$\begin{aligned} \text{iter} & : (A \rightarrow A + B) \rightarrow A \rightarrow B \\ \text{iter } f & = Y (\lambda g. (g \nabla \text{id}) \circ f) \end{aligned}$$



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Generalising Recursion Patterns

Hylomorphisms

Hylomorphisms are general recursion patterns

$$\begin{aligned} \text{hylo}_F & : (F B \rightarrow B) \rightarrow (A \rightarrow F A) \rightarrow A \rightarrow B \\ \text{hylo}_F f g & = f \circ F (\text{hylo}_F f g) \circ g \end{aligned}$$

For $\text{hylo}_F f g$

- μF recursive call tree
- g how inputs are split
- f how results are combined

map, *cata* and *ana* are just special cases of hylomorphisms

$$\begin{aligned} T A & = \mu(F A) \\ \text{map}_T f & = \text{hylo}_{F A} (\text{in}_{F B} \circ (F f \text{id})) \text{out}_{F A}, \\ & \text{where } A = \text{dom}(f) \text{ and } B = \text{codom}(f) \\ \text{cata}_F f & = \text{hylo}_F f \text{out}_F \\ \text{ana}_F f & = \text{hylo}_F \text{in}_F f \end{aligned}$$

Example: Quicksort

$$\begin{aligned} \text{split} & : \text{List } A \rightarrow T A (\text{List } A) \\ \text{split nil} & = \text{inj}_1 () \\ \text{split (cons } x l) & = \text{inj}_2 (x, \text{leq } x l, \text{gt } x l) \end{aligned}$$
$$\begin{aligned} \text{join} & : T A (\text{List } A) \rightarrow \text{List } A \\ \text{join (inj}_1 ()) & = \text{nil} \\ \text{join (inj}_2 (x, l, r)) & = l ++ \text{cons } x r \end{aligned}$$
$$\begin{aligned} \text{qsort} & : \text{List } A \rightarrow \text{List } A \\ \text{qsort} & = \text{cata}_{T A} \text{join} \circ \text{ana}_{T A} \text{split} \end{aligned}$$

or

$$\text{qsort} = \text{hylo}_{T A} \text{join split}$$



All the World's a Hylomorphism!

$$\begin{aligned} e \in E & ::= s \mid \text{par}_T p \\ s \in S & ::= f \mid e_1 \circ e_2 \mid \text{hylo}_F e_1 e_2 \\ p \in P & ::= \text{fun } s \mid p_1 \parallel p_2 \mid \text{dc}_{n,F} s_1 s_2 \mid \text{farm } n p \mid \text{fb } p \end{aligned}$$

$$\llbracket \text{par}_T p \rrbracket = \text{map}_T \mathcal{S} \llbracket p \rrbracket$$

...

$$\mathcal{S} \llbracket \text{fun } e \rrbracket = \llbracket e \rrbracket$$

$$\mathcal{S} \llbracket \text{dc}_{n,F} e_1 e_2 \rrbracket = \text{hylo}_F \llbracket e_2 \rrbracket \llbracket e_1 \rrbracket$$

...

$$\text{iter } f = \text{hylo}_{(+B)} (\text{id} \nabla \text{id}) f$$



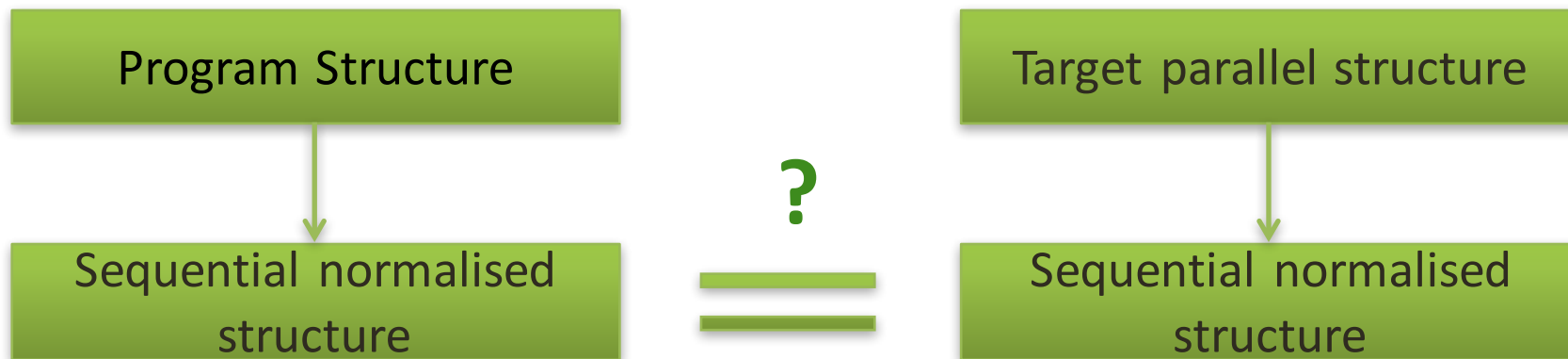
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Structure in Types



Introducing Parallel Patterns

- The type system uses a structure-equivalence relation that describes when two programs are extensionally equivalent.
- The type-checking algorithm needs to decide these structure-equivalences.



- The type-checking algorithm also needs to unify structures, modulo this structure-equivalence relation.

Syntax of Structured Types



$$e : A \xrightarrow{\sigma} \underline{B}$$

$$\begin{array}{l} \sigma \in \Sigma \\ \sigma_s \in \Sigma_s \\ \sigma_p \in \Sigma_p \end{array} ::= \begin{array}{l} \sigma_s \mid \text{PAR}_F \sigma_p \\ A \mid \sigma \circ \sigma \mid \text{HYLO}_F \sigma \sigma \\ \text{FUN } \sigma_s \mid \text{DC}_{n,F} \sigma_s \sigma_s \\ \mid \sigma_p \parallel \sigma_p \mid \text{FARM}_n \sigma_p \mid \text{FB } \sigma_p \end{array}$$

Structure-Annotated Type Rules



$$\begin{array}{c}
 \frac{\rho(f) = A \rightarrow B}{\vdash f : A \xrightarrow{A} B} \quad
 \frac{\vdash e_1 : B \xrightarrow{\sigma_1} C \quad \vdash e_2 : A \xrightarrow{\sigma_2} B}{\vdash e_1 \circ e_2 : A \xrightarrow{\sigma_1 \circ \sigma_2} C} \quad
 \frac{\vdash e_1 : F B \xrightarrow{\sigma_1} B \quad \vdash e_2 : A \xrightarrow{\sigma_2} F A \quad G = \text{base } F}{\vdash \text{hyio}_F e_1 e_2 : A \xrightarrow{\text{HYLO}_G \sigma_1 \sigma_2} B} \quad
 \frac{\vdash p : T A \xrightarrow{\sigma} T B \quad F = \text{base } T}{\vdash \text{par}_T p : T A \xrightarrow{\text{PAR}_F \sigma} T B}
 \end{array}$$

$$\frac{\vdash s : A \xrightarrow{\sigma} B}{\vdash \text{fun } s : T A \xrightarrow{\text{FUN } \sigma} T B} \quad
 \frac{\vdash s_1 : F B \xrightarrow{\sigma_1} B \quad \vdash s_2 : A \xrightarrow{\sigma_2} F A \quad G = \text{base } F}{\vdash \text{dc}_{n,F} s_1 s_2 : T A \xrightarrow{\text{DC}_{n,G} \sigma_1 \sigma_2} T B}$$

$$\frac{n : \mathbb{N} \quad \vdash p : T A \xrightarrow{\sigma} T B}{\vdash \text{farm } n p : T A \xrightarrow{\text{FARM}_n \sigma} T B} \quad
 \frac{\vdash p_1 : T A \xrightarrow{\sigma_1} T B \quad \vdash p_2 : T B \xrightarrow{\sigma_2} T C}{\vdash p_1 \parallel p_2 : T A \xrightarrow{\sigma_1 \parallel \sigma_2} T C} \quad
 \frac{\vdash p : T A \xrightarrow{\sigma} T(A+B)}{\vdash \text{fb } p : T A \xrightarrow{\text{FB } \sigma} T B}$$

Convertibility

$$\frac{\vdash e : A \xrightarrow{\sigma_1} B \quad \sigma_1 \equiv \sigma_2}{\vdash e : A \xrightarrow{\sigma_2} B}$$

$$\frac{\sigma_1 \equiv_s \sigma_2}{\sigma_1 \equiv \sigma_2} \quad \frac{\sigma_1 \equiv_p \sigma_2}{\text{PAR}_F \sigma_1 \equiv \text{PAR}_F \sigma_2}$$

$$\text{PAR}_F (\text{FUN } \sigma) \equiv \text{MAP}_F \sigma \quad (\text{PAR-EQUIV})$$

$$\text{FUN } \sigma_1 \parallel \text{FUN } \sigma_2 \equiv_p \text{FUN } (\sigma_2 \circ \sigma_1) \quad (\text{PIPE-EQUIV})$$

$$\text{DC}_{n,F} \sigma_1 \sigma_2 \equiv_p \text{FUN } (\text{HYLO}_F \sigma_1 \sigma_2) \quad (\text{DC-EQUIV})$$

$$\text{FARM}_n \sigma \equiv_p \sigma \quad (\text{FARM-EQUIV})$$

$$\text{FB}(\text{FUN } \sigma) \equiv_p \text{FUN } (\text{ITER } \sigma) \quad (\text{FB-EQUIV})$$

Plus some other rules derived from the hylomorphism laws.
We use this to produce a confluent *rewriting system*.



Parallelism Erasure

Rewrite rules derived from convertibility

$$\begin{array}{lll}
\text{FARM}_n \sigma_p & \rightsquigarrow_p & \sigma_p \\
\text{FUN } \sigma_1 \parallel \text{FUN } \sigma_2 & \rightsquigarrow_p & \text{FUN } (\sigma_1 \circ \sigma_2) \\
\text{DC}_{n,F} \sigma_1 \sigma_2 & \rightsquigarrow_p & \text{FUN } (\text{HYLO}_F \sigma_1 \sigma_2) \\
\text{FB } (\text{FUN } \sigma_1) & \rightsquigarrow_p & \text{FUN } (\text{ITER } \sigma_1) \\
\\
\text{PAR}_T (\text{FUN } \sigma_s) & \rightsquigarrow_p & \text{MAP}_T \sigma_s
\end{array}$$

Repeated to produce a confluent rewriting system

$$\begin{array}{l}
\text{erase} \quad : \quad \Sigma \rightarrow \bar{\Sigma}_s \\
\text{erase } \sigma \quad = \quad \sigma', \text{ s.t. } \sigma \rightsquigarrow_p^* \sigma' \wedge \nexists \sigma'' \text{ s.t. } \sigma'' \rightsquigarrow_p \sigma'
\end{array}$$

Normalisation

The rewrite rules are derived from basic laws

$$\begin{array}{llll}
 \text{HYLO}_F \sigma_1 \sigma_2 & \rightsquigarrow_s & \text{CATA}_F \sigma_1 \circ \text{ANA}_F \sigma_2 & \Leftarrow \sigma_1 \neq \text{IN} \wedge \sigma_2 \neq \text{OUT} & \text{(HYLO-SPLIT)} \\
 \text{CATA}_F (\sigma_1 \circ F \sigma_2) & \rightsquigarrow_s & \text{CATA}_F \sigma_1 \circ \text{MAP}_F \sigma_2 & \Leftarrow \sigma_1 \neq \text{IN} & \text{(CATA-SPLIT)} \\
 \text{ANA}_F (F \sigma_1 \circ \sigma_2) & \rightsquigarrow_s & \text{MAP}_F \sigma_1 \circ \text{ANA}_F \sigma_2 & \Leftarrow \sigma_2 \neq \text{OUT} & \text{(ANA-SPLIT)}
 \end{array}$$

...

Used to define a normalisation procedure

$$\begin{array}{ll}
 \text{norm}_s \sigma & = \sigma', \text{ s.t. } \sigma \rightsquigarrow_s^* \sigma' \wedge \nexists \sigma'' \text{ s.t. } \sigma' \rightsquigarrow_s \sigma'' \\
 \text{norm} & = \text{norm}_s \circ \text{erase}
 \end{array}$$

We can now prove equivalence of two parallel terms by:

- i) erasing parallelism using erase,*
- ii) normalising using norm, and*
- iii) testing for equivalence*



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Example

QuickSort Revisited

Start with a sequential version

$$\begin{aligned} \text{qsorts} &: \text{List}(\text{List } A) \rightarrow \text{List}(\text{List } A) \\ \text{qsorts} &= \text{map}_{\text{List}} (\text{hylo}_{F A} \text{ merge div}) \end{aligned}$$

To create a parallel divide-and-conquer version, we need to decide

$$\text{MAP}_L(\text{HYLO}_{F A A}) \cong \text{PAR}_L(\text{DC}_{n,F} A A)$$

This is easily done using a simple parallelism erasure

Inferring More Complex Parallel Structure

Now consider a more complex structure

$$\text{MAP}_L (\text{HYLO}_F A A) \cong \text{PAR}_L (\text{FARM}_n - \parallel -)$$

Normalisation of the LHS (using HYLO-SPLIT etc) gives

$$\text{MAP}_L (\text{HYLO}_F A A) \rightsquigarrow^* \text{MAP}_L (\text{CATA}_F A) \circ \text{MAP}_L (\text{ANA}_F A)$$

Parallelism erasure on the RHS gives

$$\text{PAR}_L (\text{FARM } n \ m_2 \parallel m_1) \rightsquigarrow^* \text{MAP}_L (m'_1 \circ m'_2) \\ \delta = \{m_1 \sim \text{FUN } m'_1, m_2 \sim \text{FUN } m'_2\}$$

Normalisation of the RHS gives

$$\text{MAP}_L (m'_1 \circ m'_2) \rightsquigarrow^* \text{MAP}_L m'_1 \circ \text{MAP}_L m'_2$$

Inferring More Complex Parallel Structure (2)

We need to unify the normalised forms

$$\text{MAP}_L (\text{CATA}_F A) \circ \text{MAP}_L (\text{ANA}_F A)$$

and

$$\begin{aligned} \text{MAP}_L (\text{CATA}_F A) \circ \text{MAP}_L (\text{ANA}_F A) &\sim \text{MAP}_L m'_1 \circ \text{MAP}_L m'_2 \\ \Rightarrow \Delta_1 &= \{m'_1 \sim \text{CATA}_F A, m'_2 \sim \text{ANA}_F A\} \end{aligned}$$

$$\Delta = \{\delta\} \otimes \Delta_1$$

Inferring More Complex Parallel Structure (3)

Substituting back gives us the desired parallel form

$$\text{PAR}_L (\text{FARM}_n (\text{FUN} (\text{ANA}_F A)) \parallel \text{FUN} (\text{CATA}_F A))$$

We can then use equivalence to give the actual program

$$\text{map}_{\text{List}} (\text{hylo}_{F A} \text{ merge div}) \rightsquigarrow^* \text{par}_{\text{List}} (\text{farm } n (\text{fun} (\text{ana}_{F A} \text{ div})) \parallel \text{fun} (\text{cata}_{F A} \text{ merge}))$$

Costs

For 1000 lists of 30,000,000 elements

$\text{qsorts} : \text{List}(\text{List } A) \xrightarrow{\text{min cost}} \text{List}(\text{List } A)$

$$\begin{aligned} & \text{cost} (\text{PAR}_L (\text{DC}_{n,F} A_{c_1} A_{c_2})) \text{ sz} \\ &= \max \left\{ \max_{1 \leq i \leq n} \{ c_2 (|A_{c_2}|^i \text{ sz}) \right. \\ & \quad , \text{cost} (\text{HYLO}_F A_{c_1} A_{c_2}) (|A_{c_2}|^n \text{ sz}) \\ & \quad \left. , \max_{1 \leq i \leq n} \{ c_1 (|A_{c_1}|^i |A_{c_2}|^n \text{ sz}) \} \right\} + \kappa_3(n) = 42602.72ms \end{aligned}$$

$$\begin{aligned} & \text{cost} (\text{PAR}_L (\text{FARM}_n (\text{FUN} (\text{ANA}_L A_{c_2})) \parallel (\text{FUN} (\text{CATA}_L A_{c_1})))) \text{ sz} \\ &= 27846.13ms \end{aligned}$$

$$\begin{aligned} & \text{cost} (\text{PAR}_L (\text{FARM}_n (\text{FUN} (\text{HYLO}_F A_{c_1} A_{c_2})))) \text{ sz} \\ &= 32179.77ms \end{aligned}$$



Predicted v. Actual Speedup

Pipe (Farm n_1 Func) (Farm n_2 Func)

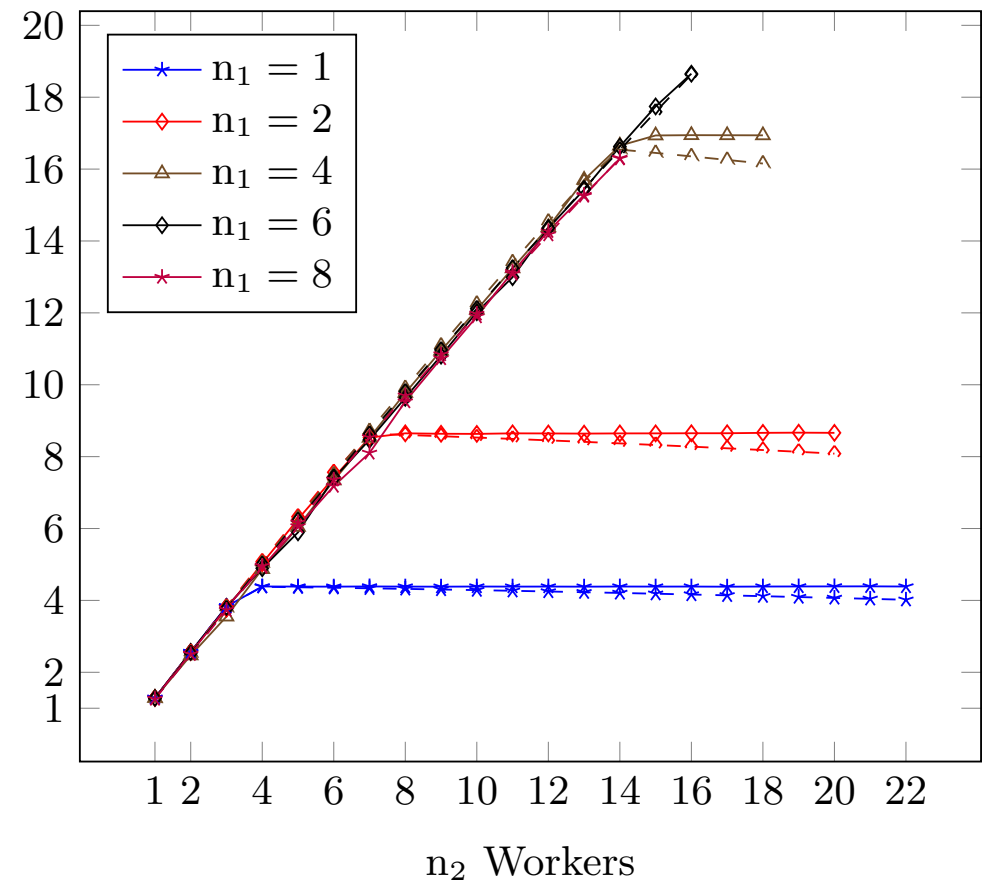
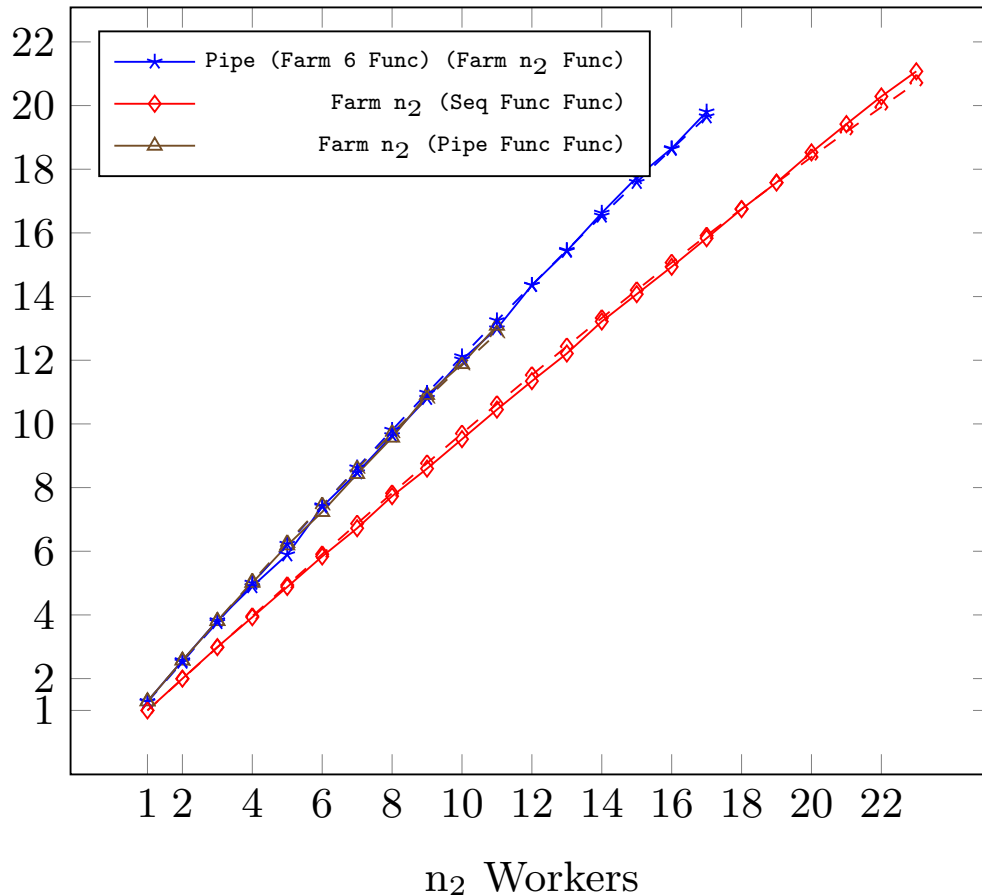


Image Convolution of 500 images on *titanic*, a 2.4GHz 24-core, AMD Opteron 6176 architecture, running Centos Linux 2.6.18-274.e15. Dashed lines are predictions.



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Conclusion

Conclusions

- First-ever treatment of parallelism that reflects parallel structure in types
- Several advantages to exposing parallel structure in types
 - **clear separation** between the structure and the functionality
 - **documentation** of how a program was parallelized
 - easy to **change the parallel structure** of a program without modifying the functional behaviour
- Reasoning about costs of different parallel structure is very powerful
 - **Automatically** find suitable parallel structures
 - Compile-time information about the run-time behaviour
 - **Automatically** rewrite programs to minimize costs

Future Work

- Other patterns, e.g. stencil and bulk synchronous parallelism
- More detailed cost models (see e.g. Hammond et al, 2016)
- Dynamic Analysis is also possible
- Allow (certain kinds of) side effects in the workers
- Implement back-ends. Run our structured programs!
- Larger case studies

Paper Available on Request



To appear in ICFP 2016

Farms, Pipes, Streams and Reforestation: Reasoning about Structured Parallel Processes using Types and Hylomorphisms

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Abstract

The increasing importance of parallelism has motivated the creation of better abstractions for writing parallel software, including structured parallelism using nested algorithmic skeletons. Such approaches provide high-level abstractions that avoid common problems, such as race conditions, and often allow strong cost models to be defined. However, choosing a *combination* of algorithmic skeletons that yields good parallel speedups for a program on some specific parallel architecture remains a difficult task. In order to

presents a new approach, a type-based mechanism that enables us to reason about the safe introduction of parallelism, while also providing a good abstraction to reason about cost. This mechanism exploits strong program structure in the form of structured parallel processes [3], combined with properties of *hylomorphisms* [22].

1.1 Motivating Example

We introduce our approach using a simple example, *image merge*, which merges pairs of images taken from an input stream. We start



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