

Adding an Extensible Backend to PQL/Java

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based on Hilmar Ackermann, Christoph Reichenbach, Christian Müller, Yannis Smaragdakis. 'A Backend Extension Mechanism for PQL/Java with Free Run-Time Optimisation', in CC'15.

- ▶ **Parallel Query Language**
- ▶ Embedded Declarative DSL as Java extension
- ▶ First-Order Logic-style queries over Java containers
- ▶ Automatic Parallelisation:
 - ▶ *Guaranteed* parallelisation (with 'right' types)

```
Set<Point> result =
```

```
    query(Set.contains(Point e)):
```

```
        s1.contains(e)
```

```
        && s2.contains(e)
```

```
        && e.x > 0;
```

- ▶ `!`, `~`, `+`, `-`, `...`, `?:`, `==`, `instanceof`, `&&`, `||`, `->`
- ▶ **forall**, **exists**
- ▶ Java expressions as constants
- ▶ `m[k]`, `m.get(k)`, `c.length`, `c.size()`,
`s.contains(e)`
- ▶ Container construction:
 - ▶ `query(Set.contains(int x))`: ...
 - ▶ `query(Array[x] == float f)`: ...
 - ▶ `query(Map.get(String s) == int i [default v])`:
...
- ▶ `reduce(sumInt) int x [over y]`: ...

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`s.contains(e)`
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 - ▶ `query(Set.contains(int x))`: ...
 - ▶ `query(Array[x] == float f)`: ...
 - ▶ `query(Map.get(String s) == int i [default v])`:
...
- ▶ `reduce(sumInt) int x [over y]`: ...

That's it — what if we need more?

- ▶ TODO:
 - ▶ Extend syntax
 - ▶ Import existing Java
 - ▶ Extend analyses / optimisations
 - ▶ Extend code generation

Making PQL Extensible

- ▶ TODO:
 - ▶ Extend syntax (*Future work*)
 - ▶ Import existing Java (*Future work*)
 - ▶ **Extend analyses / optimisations**
 - ▶ **Extend code generation**
- ▶ Sort-of WIP

PQL Translation (Static Backend)

```
import static edu.umass.pql.Query;
public class C {
public static void main(...) {
    int[] a = ...;
    Set<Point> result =
        query(Set.contains(Point e):
            s1.contains(e) && s2.contains(e)
            && e.x >= 0;
        } };
```

javac

C.class

PQL Translation (Static Backend)

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import static edu.umass.pql.Query;
public class C {
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        } };
```

javac

C.class

IL Code + Opt

```
Reduce[SET(e, r)] {
    Contains(s1, e);
    Field<POINT, x>(e, t0);
    GE<INT>(t0, 0);
    Contains(s2, e);
}
```

Query Plan

PQL Translation (Static Backend)

```
import static edu.umass.pql.Query;  
public class C {  
public static void main(...) {  
    int[] a = ...;  
    Set<Point> result =  
        query(Set.contains(Point e):  
            s1.contains(e) && s2.contains(e)  
            && e.x >= 0;  
        } }  
}
```

IL Code + Opt

javac

C.class

C\$\$PQL0.class

Backend

```
Reduce[SET(e, r)] {  
    Contains(s1, e);  
    Field<POINT, x>(e, t0);  
    GE<INT>(t0, 0);  
    Contains(s2, e);  
}
```

Query Plan

Access Modes

Contains(s_1, e)

Field($\langle \mathbf{POINT}, x \rangle$)(e, t_0)

GE($\langle \mathbf{INT} \rangle$)($t_0, 0$)

Contains(s_2, e)

foreach $e \in s_1$:

Access Modes

Contains(s_1, e)
Field(**POINT**, x)(e, t_0)
GE(**INT**)($t_0, 0$)
Contains(s_2, e)

foreach $e \in s_1$:
 $t_0 := e.x$

Access Modes

Contains(s_1, e)
Field(**POINT**, x)(e, t_0)
GE(**INT**)($t_0, 0$)
Contains(s_2, e)

foreach $e \in s_1$:
 $t_0 := e.x$
 if $t_0 \geq 0$:

Access Modes

Contains(s_1, e)
Field(**POINT**, x)(e, t_0)
GE(**INT**)($t_0, 0$)
Contains(s_2, e)

```
foreach  $e \in s_1$ :  
   $t_0 := e.x$   
  if  $t_0 \geq 0$ :  
    if  $e \in s_2$ :  
      (insert  $e$  into result set)
```

Access Modes

Contains(s_1, e)
Field(**POINT**, x)(e, t_0)
GE(**INT**)($t_0, 0$)
Contains(s_2, e)

foreach $e \in s_1$:
 $t_0 := e.x$
 if $t_0 \geq 0$:
 if $e \in s_2$:
 (insert e into result set)

Access Modes

Contains(s_1, e)
Field(**POINT**, x)(e, t_0)
GE(**INT**)($t_0, 0$)
Contains(s_2, e)

```
foreach  $e \in s_1$ :  
   $t_0 := e.x$   
  if  $t_0 \geq 0$ :  
    if  $e \in s_2$ :  
      (insert  $e$  into result set)
```

- ▶ Contains(s_1, e^w): *Write* to x (iterate over all values)
- ▶ Contains(s_1, e^r): *Read* x (contains-check)

Access Modes

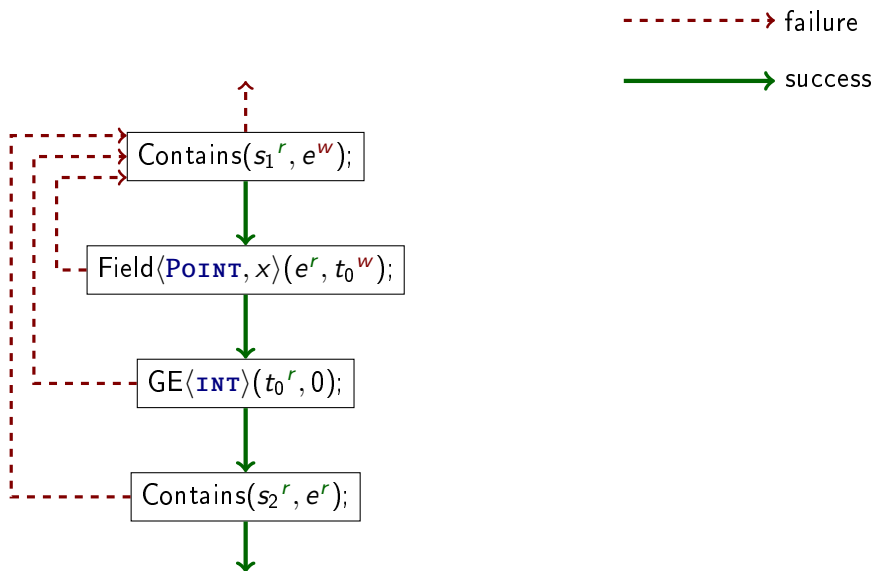
Contains(s_1^r, e^w)
Field(**POINT**, x)(e^r, t_0^w)
GE(**INT**)($t_0^r, 0$)
Contains(s_2^r, e^r)

```
foreach  $e \in s_1$ :  
   $t_0 := e.x$   
  if  $t_0 \geq 0$ :  
    if  $e \in s_2$ :  
      (insert  $e$  into result set)
```

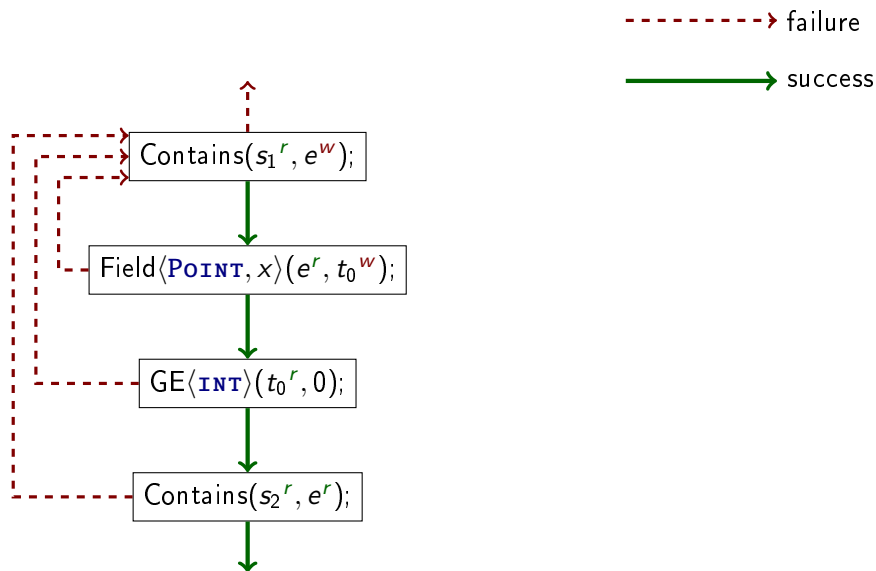
- ▶ Contains(s_1, e^w): *Write* to x (iterate over all values)
- ▶ Contains(s_1, e^r): *Read* x (contains-check)

Backend must utilise access mode information

Control Flow



Control Flow



Backend must explicitly support success/failure

Extending PQL with PQL-ESL

- ▶ PQL-ESL: PQL **E**xtension **S**pecification **L**anguage
- ▶ Describes semantics of PQL Intermediate Language operators
- ▶ Simplified Java-like syntax with some extensions and type inference

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- ▶ PQL-ESL: PQL **E**xtension **S**pecification **L**anguage
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- ▶ Simplified Java-like syntax with some extensions and type inference

```
@accessModes{rr}
```

```
ge(val1, val2) {
```

```
  local:
```

```
    if ( @type{int} val1 >= val2) proceed;
```

```
    else abort;
```

```
}
```

`GE<INT>(t0r, 0);`

Extending PQL with PQL-ESL

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Supported access modes

```
@accessModes{rr}  
ge(val1, val2) {
```

```
  local:
```

```
    if ( @type{int} val1 >= val2) proceed;  
    else abort;
```

```
}
```

```
GE<INT>(t0r, 0);
```

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Supported access modes

@accessModes{rr}

ge(val1, val2) {

local: ← Entry point label

if (@type{int} val1 >= val2) **proceed**;

else abort;

}

GE<INT>(t₀^r, 0);

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Supported access modes

```
@accessModes{rr}  
ge(val1, val2) {
```

```
  local: ← Entry point label
```

```
    if ( @type{int} val1 >= val2) proceed;  
    else abort;
```

```
}
```

Explicit type specialisation

```
GE<INT>(t0r, 0);
```


Extending PQL with PQL-ESL

- ▶ PQL-ESL: PQL **E**xtension **S**pecification **L**anguage
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Supported access modes

```
@accessModes{rr}  
ge(val1, val2) {
```

```
  local: ← Entry point label
```

```
    if ( @type{int} val1 >= val2) proceed;  
    else abort;
```

```
}
```

Explicit type specialisation

```
GE<INT>(t0r, 0);
```



Iteration in PQL-ESL

```
@accessModes{rr, rw}
contains(set, element)
{
  local:
    if (isMode( (set,element), (rw) )) {
      it = set.getIterator();
    iteration:
      hasNext = it.hasNext();
      if (hasNext == 0)
        abort;
      element = it.next();
      proceed;
    }
  }
}
```

Contains(s_1^r, e^w);

Iteration in PQL-ESL

```
@accessModes{rr, rw}
contains(set, element)
{
    Explicit access mode check
    Contains( $s_1^r, e^w$ );
    local:
        if (isMode( (set,element), (rw) )) {
            it = set.getIterator();
        iteration:
            hasNext = it.hasNext();
            if (hasNext == 0)
                abort;
            element = it.next();
            proceed;
        }
    }
}
```

Iteration in PQL-ESL

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@accessModes{rr, rw}
contains(set, element)
{
  Explicit access mode check
  Contains( $s_1^r, e^w$ );
  local:
    if (isMode( (set,element), (rw) )) {
      it = set.getIterator();
    iteration: Iteration entry point label
      hasNext = it.hasNext();
      if (hasNext == 0)
        abort;
      element = it.next();
      proceed;
    }
  }
}
```

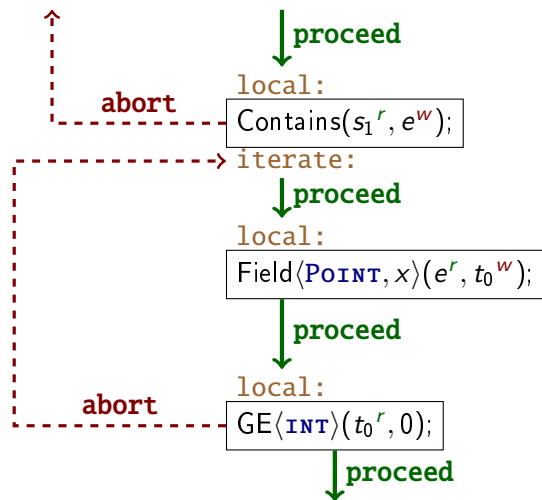
Iteration in PQL-ESL

```
@accessModes{rr, rw}
contains(set, element)
{
    Explicit access mode check Contains( $s_1^r, e^w$ );
    local:
        if (isMode( (set,element), (rw) )) {
            it = set.getIterator();
        iteration: Iteration entry point label
            hasNext = it.hasNext();
            if (hasNext == 0)
                abort;
            element = it.next();
        proceed: Assignment to parameter: reference semantics
    }
}
}
```

Iteration in PQL-ESL

```
@accessModes{rr, rw}
contains(set, element)
{
    Explicit access mode check Contains( $s_1^r, e^w$ );
    local:
        if (isMode( (set,element), (rw) )) {
            it = set.getIterator();
        iteration: Iteration entry point label
            hasNext = it.hasNext();
            if (hasNext == 0)
                abort;
            element = it.next();
            Assignment to parameter: reference semantics
        }
        if (isMode( (set,element), (rr) ) Contains( $s_2^r, e^r$ );
            tmpElement = set.contains(element);
            if (tmpElement == 0) abort;
            else proceed;
        }
    }
}
```

Control Flow



- ▶ **proceed** jumps to next `local:`
- ▶ **abort** jumps to most recent `iterate:`

Support for Linear Operators

- ▶ Linear operators are common:
 - ▶ arithmetic
 - ▶ bit operations
 - ▶ typical Java calls
- ▶ General pattern: $result = [arg0.]f(args)$
- ▶ Example: `Add<INT>(xr, yr, result)`

Support for Linear Operators

- ▶ Linear operators are common:
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 - ▶ bit operations
 - ▶ typical Java calls
- ▶ General pattern: $result = [arg0.]f(args)$
- ▶ Example: `Add<INT>(xr, yr, result)`

```
tmp = f(args);
if (isMode( (result), (r) )) {
    if (result == tmp) proceed;
    else abort;
} else {
    result = tmp;
    proceed;
}
```

Support for Linear Operators

- ▶ Linear operators are common:
 - ▶ arithmetic
 - ▶ bit operations
 - ▶ typical Java calls
- ▶ General pattern: $result = [arg0.]f(args)$
- ▶ Example: Add<INT>(x^r, y^r, result)

```
tmp = f(args);  
if (isMode( (result), (r) )) {  
    if (result == tmp) proceed;  
    else abort;  
} else {  
    result = tmp;  
    proceed;  
}
```

} **proceed on result** ?= f(args)

Templates for Types and Operators

- ▶ Parameterise types and operators

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- ▶ Parameterise types and operators
- ▶ Currently uses textual substitution

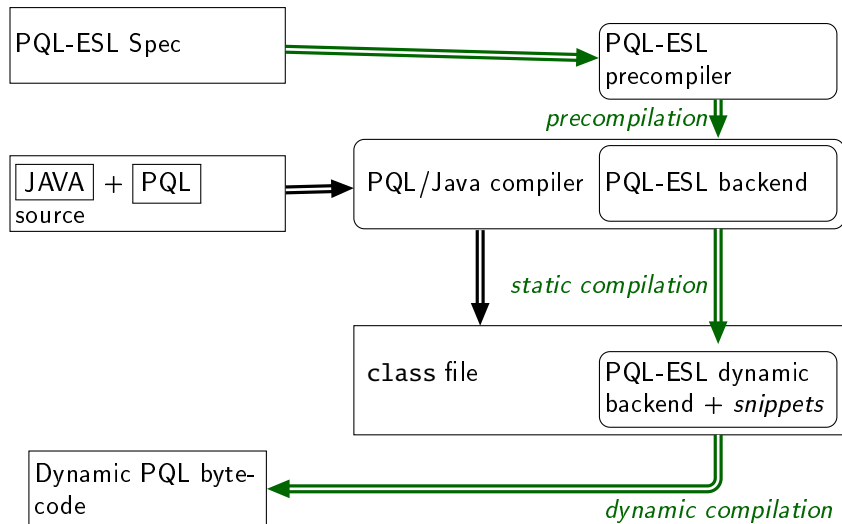
```
@generic{operator}{"<=", "<"}
@generic{type}{"int", "long", "double"}
@accessModes{rr}
lt_lte(val1, val2) {
  local:
    if ( @type{#type#} val1 #operator# val2) proceed;
    else abort;
}
```

Templates for Types and Operators

- ▶ Parameterise types and operators
- ▶ Currently uses textual substitution
- ▶ Linked to IL operators in separate step

```
@generic{operator}{"<=", "<"}  
@generic{type}{"int", "long", "double"}  
@accessModes{rr}  
lt_lte(val1, val2) {  
  local:  
    if ( @type{#type#} val1 #operator# val2) proceed;  
    else abort;  
}
```

PQL Compilation Process



snippet: family of partially linked bytecode fragments, one per IL operator

Staging and Conditionals

1. **Precompilation**
2. **Static** compilation
3. **Dynamic** compilation
4. **Execution**

Staging and Conditionals

1. **Pre**compilation
 2. **Static** compilation
 3. **Dynamic** compilation
 4. **Execution**
- ▶ Conditions evaluated as early as possible, as late as necessary

		Pre	Stat	Dyn	Exec
<code>isMode</code>	access mode	+			
<code>instanceof</code>	dynamic type check		+	+	+

Staging and Conditionals

1. **Pre**compilation
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- ▶ Conditions evaluated as early as possible, as late as necessary

		Pre	Stat	Dyn	Exec
<code>isMode</code>	access mode	+			
<code>instanceof</code>	dynamic type check		+	+	+
<code>isConst(x)</code>	<code>x</code> is constant		+	+	

Staging and Conditionals

1. **Pre**compilation
2. **Static** compilation
3. **Dynamic** compilation
4. **Execution**

- ▶ Conditions evaluated as early as possible, as late as necessary

		Pre	Stat	Dyn	Exec
<code>isMode</code>	access mode	+			
<code>instanceof</code>	dynamic type check		+	+	+
<code>isConst(x)</code>	<code>x</code> is constant		+	+	
<code>isParallel()</code>	parallel execution			+	

- ▶ `isParallel` occurs in code
⇒ PQL assumes that the operator supports parallelisation

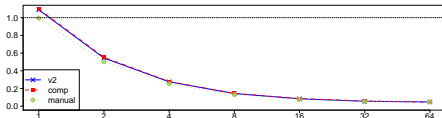
- ▶ Basic Java
- ▶ Genericity
- ▶ **proceed** and **abort**
- ▶ **proceed on**
- ▶ Access to static analysis
- ▶ Staging
- ▶ Other features:
 - ▶ Parallel execution
 - ▶ Custom Rewriting (WIP)
 - ▶ Cost model
 - ▶ Once-only Precomputation (**global:**)

- ▶ Re-implemented PQL backend using PQL-ESL
- ▶ Added support for dynamic compilation
- ▶ Added extensions:
 - ▶ `sqrt`
 - ▶ `modulo`
 - ▶ `isPrime(n)` and `primesRange(min, x, max)`
 - ▶ Java 8 Streams
 - ▶ SQL
- ▶ Added rewriting:
 - ▶ `isPrime + range` \Rightarrow `primesRange`
 - ▶ PQL \Rightarrow DB access operators
etc.

General Performance

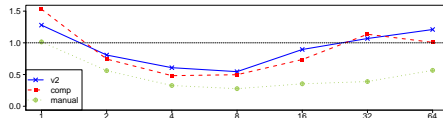
wordcount

normalized to 3330.720855ms



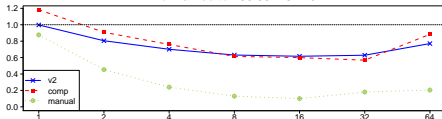
thregrep

normalized to 22.831667ms



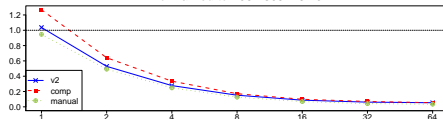
bonus

normalized to 250.991431ms

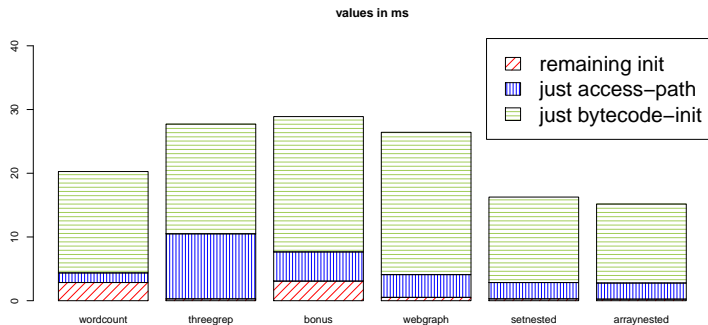


webgraph

normalized to 1634.369175ms



Overhead



Dynamic compilation comes at a price

Dynamic Optimisation

```
import static edu.umass.pql.Query;
public class C {
public static void main(...) {
    int[] a = ...;
    Set<Point> result =
        query(Set.contains(Point e):
            s1.contains(e) && s2.contains(e)
            && e.x >= 0;
        )
} }
```

javac

C.class

C\$\$PQL0.class

Backend

```
Reduce[SET( $e^r, r^w$ )] {
    Contains( $s_1^r, e^w$ );
    Field<POINT, x>(er, t0w);
    GE<INT>(t0r, 0);
    Contains( $s_2^r, e^r$ ); }
```

Dynamic Optimisation

```
import static edu.umass.pql.Query;
public class C {
public static void main(...) {
    int[] a = ...;
    Set<Point> result =
    query(Set.contains(Point e):
        s1.contains(e) && s2.contains(e)
        && e.x >= 0;
    }
}
```

javac

C.class

dynamic bytecode

Frontend

Unoptimised

```
Reduce[SET(e, r)] {
    Type<POINT>(e);
    Contains(s1, e);
    Contains(s2, e);
    Field<POINT, x>(e, t0);
    GE<INT>(t0, 0); }
```

Opt

Opt

```
Reduce[SET(er, rw)] {
    Contains(s1r, ew);
    Field<POINT, x>(er, t0w);
    GE<INT>(t0r, 0);
    Contains(s2r, er); }
```

Plan 1

```
Reduce[SET(er, rw)] {
    Contains(s2r, ew);
    Field<POINT, x>(er, t0w);
    GE<INT>(t0r, 0);
    Contains(s1r, er); }
```

Plan 2

- ▶ Compare different execution orders

Dynamic Optimisation

```
import static edu.umass.pql.Query;  
public class C {  
public static void main(...) {  
    int[] a = ...;  
    Set<Point> result =  
        query(Set.contains(Point e):  
            s1.contains(e) && s2.contains(e)  
            && e.x >= 0;  
        } }  
}
```

javac

C.class

dynamic bytecode

Frontend

Unoptimised

```
Reduce[SET(e, r)] {  
    Type<POINT>(e);  
    Contains(s1, e);  
    Contains(s2, e);  
    Field<POINT, x>(e, t0);  
    GE<INT>(t0, 0); }  
}
```

Opt

Opt

```
Reduce[SET( $e^r, r^w$ )] {  
    Contains( $s_1^r, e^w$ );  
    Field<POINT, x>(er, t0w);  
    GE<INT>(t0r, 0);  
    Contains( $s_2^r, e^r$ ); }  
}
```

Plan 1

```
Reduce[SET( $e^r, r^w$ )] {  
    Contains( $s_2^r, e^w$ );  
    Field<POINT, x>(er, t0w);  
    GE<INT>(t0r, 0);  
    Contains( $s_1^r, e^r$ ); }  
}
```

Plan 2

- ▶ Compare different execution orders
- ▶ Here: Iterate over s_1 or s_2 ?

Dynamic Optimisation: Extreme Cases

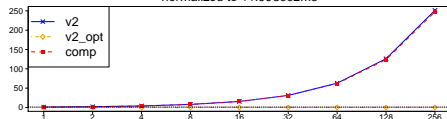
```
Set<Point> result =  
    query(Set.contains(Point e)):  
        s1.contains(e)  
        && s2.contains(e)  
        && e.x > 0;
```

Dynamic Optimisation: Extreme Cases

```
Set<Point> result =  
    query(Set.contains(Point e):  
        s1.contains(e)  
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```

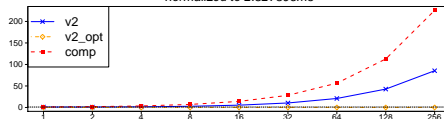
setnested

normalized to 11.993002ms



arraynested

normalized to 2.827396ms

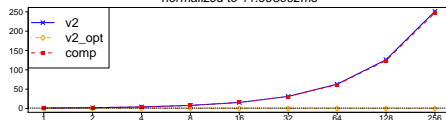


Dynamic Optimisation: Extreme Cases

```
Set<Point> result =  
    query(Set.contains(Point e):  
        s1.contains(e)  
        && s2.contains(e)  
        && e.x > 0;
```

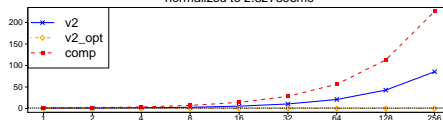
setnested

normalized to 11.993002ms



arraynested

normalized to 2.827396ms



Static compilation can't know whether s1 or s2 is bigger, can't optimise

Conclusions

- ▶ DSL for backend extensions:
 - ▶ Compact
 - ▶ Permits staged evaluation
- ▶ PQL backend rewrite:
 - ▶ Enabled dynamic optimisation
 - ▶ Dramatically faster whenever dynamic knowledge helps
 - ▶ Competitive when not
 - ▶ Dynamic recompilation time too high: \Rightarrow JIT?
- ▶ Next steps:
 - ▶ New frontend (JastAdd)
 - ▶ Make PQL-ESL easier to use
 - ▶ Eliminate labels
 - ▶ Directly migrate Java bytecode to PQL-ESL
 - ▶ Easy-to-use rewriting formalism

Extension Example: SQL Access

```
DB(spec, db);  
Table(db, "tblname", tbl);  
Column(tbl, "column", c);  
GT(c, 0);  
Contains(set, c);
```

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```
SELECT * FROM tblname;
```

```
transfer ResultSet
```

```
filter: {c | v ∈ ResultSet,  
          c = v.column,  
          c > 0, c ∈ set}
```

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```

```
DB(spec, db);  
TableQuery(db, "tblname", ["column"],  
           [DBGT("column", 0)], tbl);  
Column(tbl, "column", c)  
Contains(set, c)
```

```
SELECT * FROM tblname;
```

transfer *ResultSet*

```
filter: {c | v ∈ ResultSet,  
           c = v.column,  
           c > 0, c ∈ set}
```

Rewriting Support (WIP)

Extension Example: SQL Access

<code>DB(<i>spec</i>, <i>db</i>);</code>	<code>DB(<i>spec</i>, <i>db</i>);</code>
<code>Table(<i>db</i>, "<i>tblname</i>", <i>tbl</i>);</code>	<code>TableQuery(<i>db</i>, "<i>tblname</i>", ["<i>column</i>"],</code>
<code>Column(<i>tbl</i>, "<i>column</i>", <i>c</i>);</code>	<code> [DBGT("<i>column</i>", 0)], <i>tbl</i>);</code>
<code>GT(<i>c</i>, 0);</code>	<code>Column(<i>tbl</i>, "<i>column</i>", <i>c</i>)</code>
<code>Contains(<i>set</i>, <i>c</i>);</code>	<code>Contains(<i>set</i>, <i>c</i>)</code>

<code>SELECT * FROM <i>tblname</i>;</code>	<code>SELECT <i>column</i> FROM <i>tblname</i> WHERE <i>column</i> > 0;</code>
<code>transfer <i>ResultSet</i></code>	<code>transfer <i>ResultSet</i></code>
<code>filter: {<i>c</i> <i>v</i> ∈ <i>ResultSet</i>, <i>c</i> = <i>v.column</i>, <i>c</i> > 0, <i>c</i> ∈ <i>set</i>}</code>	<code>filter: {<i>c</i> <i>c</i> ∈ <i>ResultSet</i>, <i>c</i> ∈ <i>set</i>}</code>

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```
DB(spec, db);  
TableQuery(db, "tblname", ["column"],  
           [DBGT("column", 0)], tbl);  
Column(tbl, "column", c)  
Contains(set, c)
```

```
SELECT * FROM tblname;
```

transfer ResultSet

```
filter: {c | v ∈ ResultSet,  
         c = v.column,  
         c > 0, c ∈ set}
```

```
SELECT column FROM tblname WHERE column > 0;
```

transfer ResultSet

```
filter: {c | c ∈ ResultSet, c ∈ set}
```

Rewriting Support (WIP)

Query Planning Strategy

- ▶ Dynamic Programming selects shortest path
- ▶ Cost model:
 - ▶ *cost*: how much does one option cost?
 - ▶ *size*: how many options will we iterate over?
 - ▶ *selectivity*: how likely will past bindings match?
 - ▶ *parallel*: is this relation parallelisable?
 - ⇒ discounts *size* when applicable

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- ▶ Dynamic Programming selects shortest path
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 - ▶ *parallel*: is this relation parallelisable?
 - ⇒ discounts *size* when applicable
- ▶ *c*: aggregate cost
- ▶ *s*: aggregate size and selectivity

$$c' = c + \text{cost} \times s$$

$$s' = s' \times \text{size} \times \text{selectivity}$$

Predicting execution cost

```
cost_formula:
    formula = "1.0";
    __cost_formula = formula;

proceed_formula: // = size × selectivity
    if (isMode( (obj, key), (rr) )) {
        if ( isMode( (value), (r) ) )
            formula = "0.004";
        else formula = "1.0";
    } else {
        if (isConst(obj)) {
            if (isMode( (obj, key, value), (rwr || r_r) ))
                formula = "obj[size]/250";
            else formula = "obj[size]";
        } else {
            if (isMode( (obj, key, value), (rwr || r_r) ))
                formula = "100/250";
            else formula = "100";
        }
    }
    __proceed_formula = formula;
```